PRECISION MEASUREMENT OF THE NEUTRON LIFETIME
WITH THE UCN$\tau$ EXPERIMENT

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Because it lacks an electric charge, the neutron is a useful probe of the electroweak framework of the Standard Model (SM), which describes the process as a particle undergoes radioactive decay. Neutrons are the simplest example of $\beta$-decay, which provides a unique suite of tests for fundamental parameters of electroweak theory. The neutron lifetime, $\tau_n$, can be used in conjunction with its other decay properties to extract $V_{ud}$. This can then be combined with other particle decays to investigate the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. By itself, neutron $\beta$-decay can be used to study the origins of matter in the universe, and even hunt for new novel decay modes. The UCN$\tau$ experiment, at Los Alamos National Laboratory (LANL), is the world’s most precise measurement of the neutron lifetime. UCN$\tau$ traps Ultracold Neutrons (UCN) in a bottle of permanent magnets for periods of time longer than the neutron lifetime. This work presents the results of two calendar years of UCN$\tau$ data taking, resulting in a lifetime with both statistical and systematic uncertainties below 0.3 s. The analysis has utilized new methods to minimize the uncertainties due to backgrounds and initial estimation of the number of trapped UCN. A suite of Monte Carlo simulations, in conjunction with data-driven measurements, have been used to demonstrate the loss rates of UCN in UCN$\tau$ are below the statistical precision of the experiment. This analysis leads to a blinded lifetime result with uncertainties on $\tau_n$ of $\pm 0.26^{+0.21}_{-0.16}$ s, or a relative precision of $4 \times 10^{-4}$. 

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CHAPTER 1
INTRODUCTION

1.1 NEUTRON OVERVIEW

The neutron, $n$, is a chargeless, massive nucleon originally theorized by Rutherford in 1920[2]. Atomic experiments of the time had shown the existence of tightly packed nuclei at the core of atoms. A neutral particle consisting of a tightly bound proton and electron was posited to explain the structure of various isotopes. Later, in 1932, Chadwick provided the first experimental evidence for the neutron[3]. Previous experiments had bombarded $^9$Be isotopes with $\alpha$-particles, a reaction now recognized as:

$$^9\text{Be} + ^4\text{He} \rightarrow ^{12}\text{C} + n.$$ (1.1)

Chadwick explained these results using a neutral particle with a mass comparable to the proton.

The presence of an additional neutral particle allowed for energy and momentum conservation. Subsequent experiments by Chadwick and Goldhaber showed that the neutron mass was too high to be explained as a bound state of the proton and the electron[4]. Based upon this mass discrepancy, the neutron was considered to be unstable. Around this time, Fermi had published a model for nuclear $\beta$-decay[5]. In $\beta$-decay, a neutron decays into a proton, $p$, an electron, $e$, and an antineutrino, $\bar{\nu}_e$, through the process:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e.$$ (1.2)

Modern particle physics describes $\beta$-decay using the Standard Model (SM). The neutron consists of three valence quarks; one up quark $u$ and two down quarks $d$ combine
Figure 1.1: Feynman Diagram showing the decay of the free neutron. The valence quarks \( uud \) for a proton and \( udd \) for a neutron can be seen in the initial and final state. The weak \( W \) boson mediates the transition from a valence \( d \) quark into an \( u \) quark.

to form a neutron, denoted as \( udd \). In this weak transition, one of these \( d \) quarks transforms to an \( u \) quark, forming a proton, denoted as \( uud \). This interaction can be seen in figure 1.1. The SM provides a means to directly calculate the decay rates of particles. Excluding higher order terms, the differential decay rate of a polarized neutron can be written as[6, 7, 8, 9]:

\[
\frac{d\Gamma^3}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2 |V_{ud}|^2}{(2\pi)^5} |p_e| E_e E_v^2 (1 + 3\lambda^2) \times \left( 1 + b \frac{m_e}{E_e} + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_v} + A \frac{\vec{\sigma} \cdot \vec{p}_e}{E_e} + B \frac{\vec{\sigma} \cdot \vec{p}_\nu}{E_v} \right).
\]  

Here, the decay rate, \( \Gamma \), over the solid angles of momentum of the resultant particles, \( \Omega_e, \Omega_\nu \), depends on the energy and momenta of the resultant particles, \( E_e, E_v, \vec{p}_e, \) and \( \vec{p}_\nu \), as well as the spin of the neutron \( \vec{\sigma} \). Additionally, this incorporates the Fermi constant, \( G_F \), which must be determined through muon decay[10]. A set of “correlation coefficients” \( a, A, B, \) and \( b \), as well as some higher-order terms excluded in equation (1.3), can be probed by looking at decay asymmetries. These asymmetries overconstrain the free parameters of the SM: \( \lambda \) and \( V_{ud} \). Form factors relating the axial-vector component, \( g_A \), and vector component, \( g_V \) of this charged-current interaction have been absorbed into
one value $\lambda = g_A / g_V$.\footnote{The $\lambda$ written here can technically be complex: $\lambda = |g_A| / |g_V| e^{i\phi}$. Under $T$-reversal invariance, the phase $\phi = \pi$. Furthermore, the decay rate equation (1.3) assumes a Conserved Vector Current (CVC), such that $g_V = 1$.} The Cabibbo Kobayashi Maskawa (CKM) matrix, of which $V_{ud}$ is an element, describes the transition probability between quark flavor eigenstates\cite{11}:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
=
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}.
$$

(1.4)

Integrating equation (1.3) over available phase space provides a direct relationship between the neutron lifetime, $\tau_n$, and the aforementioned parameters, $V_{ud}$ and $\lambda$\cite{12}:

$$
\tau_n^{-1} = \frac{m_e^5 c^4}{2 \pi^3 \hbar^2} G_F^2 |V_{ud}|^2 \left( 1 + 3 \lambda^2 \right) f (1 + \delta_R).
$$

(1.5)

This calculation requires the knowledge of the phase-space availability of this decay, $f$, as well as the nature of some higher-order corrections, $\delta_R$. These terms, in addition to equation (1.5), will be expanded upon in section 1.3.2.

This work primarily focuses on the neutron lifetime, $\tau_n$, describing in detail one precision measurement of $\tau_n$. The remainder of this chapter, as well as chapter 2, will instead describe previous theoretical and experimental works. Further discussion of general properties of low-energy neutrons can be found in section 1.2. Following this will be an investigation of physics applications of precision neutron decay measurements in section 1.3. This section will also detail much of the present theoretical underpinning of precision $\beta$-decay measurements.
1.2 UCN OVERVIEW

1.2.1 Introduction

By a useful coincidence of nature, the neutron can be trapped by magnetic, gravitational, and material forces of roughly the same energy[13]. This provides a mechanism by which low energy neutrons can be stored in dedicated containers. Such low-energy, trappable, Ultra Cold Neutrons (UCN) can then be used for various fundamental physics tests. Tautologically, a UCN is a neutron with kinetic energy low enough to be trapped by these various fields and material potentials. Depending on the material used to construct the trap, this occurs at a kinetic energy around 300 neV. The temperature of an ideal gas can be related to its energy by \( E = \frac{3}{2}k_B T \), and so an “ideal gas” of UCNs has a temperature of \( T \sim 2.3 \) mK.

The fields capable of trapping UCN will be discussed in section 1.2.2. These include both magnetic and gravitational fields. Material interactions, in particular the trapping ability of various materials, will be described in section 1.2.3. The combination of these two methods of UCN storage can then be used as a tool to construct experiments designed to probe neutron decays.

1.2.2 Neutrons in Fields

The neutron has a mass of \( m_n = 1.0087 \) amu[14]. Acceleration due to earth’s gravity does not depend on the mass of the particle. Typically, subatomic particles have a high enough energy that the influence of this force is negligible. UCN however have low enough kinetic energy that gravity actually affects its trajectory. The gain in potential from a neutron rising a distance \( h \) in the earth’s gravitational field, \( g = 9.8 \) m/s\(^2\), can be simply written as:

\[
E = m_n gh.
\]  

(1.6)

For the neutron, this corresponds to an energy gain of 102 neV / m.
The neutron has a magnetic dipole moment, $\mu_n = -1.913 \mu_N$, due to being a spin-$\frac{1}{2}$ particle. This has two effects. Low-energy neutrons can be polarized by either material interactions or magnetic fields, which can be used to probe the internal structure of materials[15]. This has many condensed matter physics applications, as the lack of charge allows neutrons to penetrate deep and probe the magnetic structures of crystals. More relevant for this work, this magnetic dipole moment provides an additional physics mechanism for trapping UCN. The neutron can be treated as an ideal magnetic dipole, with a potential energy due to an external magnetic field, $\vec{B}$:

$$E = -\vec{\mu}_n \cdot \vec{B}. \quad (1.7)$$

A neutron moving into a 1 T magnetic field will thus gain $\sim 60$ neV in energy. For UCN the adiabatic condition applies:

$$\frac{1}{|\vec{B}|} \left| \frac{d\vec{B}}{dt} \right| \ll \frac{\vec{\mu}_n \cdot \vec{B}}{\hbar}. \quad (1.8)$$

This means that trapped UCN will maintain its spin vector in the direction of the magnetic field[13]. Violations of the adiabatic condition could lead to neutron depolarization, which leads to an important systematic uncertainty in magnetic storage experiments.

The SM predicts that the neutron has a very small Electric Dipole Moment (EDM). Various theories of physics beyond the SM include an increased neutron EDM. Additionally, a non-zero neutron EDM could help explain the matter-antimatter asymmetry in the universe. Numerous recent experiments are underway to find a non-zero electric dipole moment[16, 17]. These experiments look for variations in the precession frequency of neutrons in a magnetic field, with an external electric field that flips polarity. However, the neutron EDM is many orders of magnitude lower than the magnetic dipole moment, and thus electrical trapping of neutrons is not practical.
1.2.3 UCN Interactions with Matter

Neutrons also interact with surface materials via the strong interaction. A neutron, with de Broglie wavelength $\lambda_n$, scattering off a single spherical nucleus, with scattering length $a$, can be described by a plane wave plus some scattered wave. The Schrödinger equation for such a neutron located at $r$ scattering off a nucleus at $r_n$ can then be assumed to have a potential described by a step function:

$$\frac{-\hbar^2}{2\mu} \nabla^2 \psi + [E - U(\eta)] \psi = 0 \quad \begin{cases} 
U(\eta) = -U_0 & \eta < \rho \\
U(\eta) = 0 & \eta > \rho
\end{cases} \quad (1.9)$$

Here we have introduced the reduced mass $\mu$ and the relative motion $\eta = r - r_n$, as well as an effective radius $\rho$, which satisfies $a \ll \rho \ll \lambda_n$. Because UCN have low energies, the “scattered wavefunction” of this interaction only sees the $s$-wave contribution from these nuclei. Then the step potential $U$ can be written as[13]:

$$U_F(\eta) = \frac{2\pi \hbar^2 a}{\mu} \delta^{(3)}(\eta). \quad (1.10)$$

The solution for $U$ can be found by using the Born approximation. For real surfaces, equation (1.10) can be summed over all atoms of interest, which are located at $r_i$ and have a bound nucleus scattering length of $a_i$:

$$E_f(r) = \frac{2\pi \hbar^2}{m} \sum_i a_i \delta(r - r_i). \quad (1.11)$$

This sum can be volume averaged over the entire material to form an effective potential. The “Fermi Potential” described by averaging $a_i$ in equation (1.11) over all nuclei in the surface is tabulated for individual materials. This potential can then be treated as a 1-dimensional step potential at the material surface. This can be solved as a typical quantum mechanical interaction, and the reflected and transmitted wave amplitudes can
be found analytically. The reflected wave amplitude, $R$ can be shown to be:

$$R = \frac{(E_\perp)^{1/2} - (E_\perp - E_f)^{1/2}}{(E_\perp)^{1/2} + (E_\perp - E_f)^{1/2}}. \quad (1.12)$$

For a UCN with incident energy $E_\perp < E_f$, the amplitude of the reflected wave $|R|^2 = 1$.

A small component of the neutron’s wavefunction enters the classically forbidden region of a step potential. In the event of an imaginary component of the Fermi Potential, $U = E_f - iW$, a UCN with $E_\perp < E_f$ can still be lost. In the case that the imaginary component $W \ll E_f$, the reflection off a 1-dimensional surface described in equation (1.12) has an additional component:

$$|R|^2 = 1 - 2 \frac{W}{E_f} \left( \frac{E_\perp}{E_f - E_\perp} \right)^{1/2}. \quad (1.13)$$

This loss per bounce, even in otherwise below-threshold UCN conditions, provides an incentive to avoid the use of material storage for UCN lifetime experiments. Material storage of UCN will always result in some loss every time a UCN interacts with the surface.

1.3 WHY STUDY NEUTRON DECAY

1.3.1 Theoretical Motivation

Precision measurements of the neutron lifetime provide a useful tool for testing the standard model. While much physics can be done at high-energy accelerators, low energy precision measurements complement these experiments, probing different regions of parameter space. The neutron, in particular, provides a useful probe of the SM. Neutrons are the only nuclear decay without complicated additional form factors resulting from interactions between nucleons. Additionally, neutrons are the lightest known hadronic decay involving three valence quarks. Finally, neutrons are readily available at research
centers throughout the globe, providing the ability for consistency checks. These reasons combine to make neutrons an attractive prospect for studying physics beyond the standard model.

Precision measurements of neutron decay allow for studies of assorted SM processes. In particular, recent studies of CKM matrix unitarity have raised potential tension with the standard model. This will be discussed in section 1.3.2. At present, there is a “neutron lifetime discrepancy” between methods of measuring the lifetime. Section 1.3.3 will focus on the potential non-experimental resolutions of this discrepancy. The neutron lifetime in particular provides specific inputs for models of evolution of the early universe, which will be described further in section 1.3.4.

1.3.2 CKM Unitarity

The CKM matrix of equation (1.4) characterizes the electroweak mixing between quark flavor eigenstates. As this matrix describes a probability, each quark should have a total transition probability of 1. Looking just at the first row:

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta, \tag{1.14}
\]

the CKM matrix is unitary if \( \Delta = 0 \). In the event that \( \Delta \neq 0 \), this is a direct measurement of new physics, requiring the existence of additional quarks or other transitions. Various modifications to the SM can lead to deviations from unitarity, and so precision measurements of the CKM matrix can elucidate additional avenues for theoretical and experimental research[18].

Measurements of \( V_{us} \) and \( V_{ub} \) come from the decays of particles other than neutrons. The most precise measurement of \( V_{us} \) comes from semi-leptonic kaon decays. This has a value of \( |V_{us}| = 0.2245(8)[14] \). \( V_{ub} \) can be measured in b-mesons, but has a value of \( |V_{ub}| \sim 10^{-5} \). At the level of precision in \( V_{ud} \) and \( V_{us} \), the matrix element \( V_{ub} \) is thus
negligible in the unitarity calculation.

In addition to measurements of $V_{ud}$ from neutron decays, $0^+ \rightarrow 0^+$ “superallowed” nuclear decays provide stringent constraints on $V_{ud}$. These specific nuclear $\beta$-decays both begin and end in the nuclear state spin state $J^\pi = 0^+$ and the isospin state $T = 1$. Since the axial vector component does not contribute, and by the conserved vector current (CVC) hypothesis $g_V = 1$, these nuclei share a constant $f_t$ value[19]:

$$f_t = \frac{K}{2 (G_F V_{ud})^2} = \text{const}, \quad (1.15)$$

where $K/(\hbar c)^2 = 2\pi^3 \hbar l \ln (2)/(m_e c^2)^5$ is a constant. Equation (1.15) is not exactly correct. Due to shifts in radiative corrections, $\delta_R$, and small isospin symmetry breaking, $\delta_C$, the $f_t$ value must be amended slightly. This takes the form of a “corrected” $F_t$ value:

$$F_t \equiv f_t (1 + \delta_R) (1 - \delta_C) = \frac{K}{2 (G_F V_{ud})^2 (1 + \Delta V_R)} = \text{const}. \quad (1.16)$$

For each $0^+ \rightarrow 0^+$ decay, the $f_t$ value can be experimentally measured to a high precision, of $O(0.1)\%$. However, these decays might not all end up in the $0^+$ state, and so the branching ratios for $0^+ \rightarrow 0^+$ must also be measured to a high precision as well. These can then be fit in conjunction with theoretically derived radiative correction values to find the constant $F_t$. Finally, this fit can be done across many isotopes to experimentally determine the value of $V_{ud}$[20]:

$$|V_{ud}|^2 = \frac{2984.43 \, \text{s}}{F_t (1 + \Delta V_R)}. \quad (1.17)$$

The limiting factors for these nuclear decays are the theoretical nuclear structure components and the radiative decay corrections to the decay. Neutron decay has no nuclear structure correction functions, but are still subject to radiative corrections in the decay process. Solving numerically for $V_{ud}$, the analogous relationship for the neutron
\[ |V_{ud}|^2 = \frac{5099.3 \text{ s}}{\tau_n (1 + 3\lambda^2) (1 + \Delta_R)}. \] (1.18)

The radiative corrections for both equation (1.17) and equation (1.18) feature the same nucleus-independent “inner” contribution\[22\]. However, the two methods differ in their treatment of an “outer” correction that is nucleus-dependent. Recent theoretical work on these radiative corrections has brought tension between the \(V_{ud}\) value from superallowed decays and unitarity\[23, 24\]. An improved measurement of the neutron lifetime at the 0.3 s level, in conjunction with improved measurements of \(\lambda\), would allow for a unitarity test that does not rely upon nuclear structure functions.

Additional complementary measurements of \(V_{ud}\) can come from pion decay. These measurements determine the branching ratio of rare pion \(\beta\)-decay, \(\pi^+ \rightarrow \pi^0 + e^+ + \nu_e\)[25]. However, these measurements are not yet at the required precision to provide a competitive determinations of \(V_{ud}\). These are nevertheless attractive due to their theoretical simplicity, lacking any nuclear corrections and requiring smaller radiative corrections than the neutron\[26\].

The present state of the experimental values of \(V_{ud}\) unitarity is shown in figure 1.2. While at present neutron decay experiments lack the precision to test unitarity, future experimental improvements can provide such a limit.

### 1.3.3 Exotic Neutron Decays

A lifetime experiment seeks to understand an exponential decay process. Two general methods of measuring a neutron lifetime exist. An experiment can either count the “living” particles after some amount of time, or count the “dead” or decayed particles, typically by measuring the decay products. For the neutron, these two methods mean either counting neutrons in a “bottle” or counting decay product protons or electrons from a neutron “beam.” Many of these experiments have been done; an overview will be found in chapter 2. At present there is an 8.7 s discrepancy between the two methods,
Figure 1.2: Experimental methods of calculating $V_{ud}$. The horizontal bars indicate other non-neutronic methods of calculating $V_{ud}$, while the vertical and diagonal bars indicate experimental values from neutron decay. The diagonal bars include both the “bottle” and “beam” methods of calculating the neutron lifetime. The $0^+ \rightarrow 0^+$ determination of $V_{ud}$ disagrees with the expected value from experimental values of $V_{us}$, assuming unitarity.

Corresponding to $\sim 4\sigma$[27]. This could be indicative of a new decay channel for the neutron.

In addition to neutrons decaying via equation (1.2), there could be other other processes. A “radiative decay” of the neutron is a standard model process that includes a continuous photon spectrum[28]. This is caused by the emission of a bremsstrahlung $\gamma$ with energy $< 782$ keV, typically from the $e^-$, in addition to the normally produced products:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e + \gamma.$$  \hspace{1cm} (1.19)

This branching ratio is $BR = (9.17 \pm 0.24 \pm 0.64) \times 10^{-3}$[29]. Occasionally, the resultant
\[ e^- \] lacks the energy to escape the \[ p^+ \], instead forming a neutral Hydrogen atom, \[ n \rightarrow H + \nu_e \] [30]. This branching ratio is even smaller than the radiative decay, \[ BR < 2.7 \times 10^{-3} \] [31]. These decays are known via the SM, and thus should not contribute to the potential discrepancy.

Beyond the standard model, exotic decays would potentially lead to some level of symmetry breaking, either violating baryon number or charge conservation. The neutron, as a neutral particle, could have some small non-zero Majorana mass. This would allow the neutron to oscillate into an antineutron, \[ n \rightarrow \bar{n} \] [32]. Such a transition, if possible, would create a \[ \Delta B = 2 \] baryon-number transition. This is a potential explanation for the matter-antimatter discrepancy in the universe by providing a mechanism for Baryogenesis.

Rather than oscillating to a known SM particle, \( \bar{n} \), the neutron could instead probe the existence of a “mirror” sector [33]. This theory postulates the existence of a duplicate of the SM, only with the opposite parity particles and interactions. This then would suggest the presence of a “mirror neutron,” \( n' \), with the same mass, \( m_n \), as the regular neutron. Due to the presence of the Earth’s magnetic field, the regular and mirror neutrons would feel different potentials, \( V \) and \( V' \) respectively. In the event of a small mixing element, \( \delta m \), between the two sectors, a Hamiltonian describing the transition between the two can be written as:

\[
\mathcal{H} = \begin{pmatrix}
    m_n - i/2\tau_n - V & \delta m \\
    \delta m & m_n - i/2\tau_n - V'
\end{pmatrix}.
\]

Various experimental efforts to determine \( \delta m \) have begun [34, 16].

Finally, neutrons could additionally decay to some other dark matter particle [35]. Such a dark decay is limited by information from other nuclear decays, of which the most stringent is \( ^9\text{Be} \). Based on these limits, the output energy, \( M_f \), from such a dark decay must be between \( 937.900 \text{ MeV} \leq M_f \leq 939.565 \text{ MeV} \). These dark decays can be
probed by looking at potential additional decay products. Limits have been placed on
dark decays also involving an electron-positron pair or an extra $\gamma$[36, 37]. By improving
the uncertainty limits on $\tau_n$, additional limits can be placed on such exotic processes
beyond the SM.

1.3.4 Primordial Helium Abundance

The neutron plays an important role in modeling of the early universe in Big Bang
Nucleosynthesis (BBN)[38]. Statistical mechanical models of the early universe use the
baryon-to-photon ratio as an input. Early on in the universe, matter was hot and dense
enough to sustain the reactions $n + \nu_e \leftrightarrow p^+ + e^-$ and $n + e^+ \leftrightarrow p^+ + \nu_e$, in addition to
neutron $\beta$-decay. At some point, the universe expanded enough such that the weak inter-
action rate fell below the Hubble expansion rate. After this “freezeout” temperature, the
number of nucleons became fixed. Most neutrons will be stored in the tightly bound $^4$He
nucleus, in what is known as “Primordial Helium,” formed during the nucleosynthesis
phase of the early universe.

The mass fraction of primordial $^4$He, $Y_p$, depends on the relative baryon abundance
in the early universe, $\Omega_b$, and the ratio of protons to neutrons at the freezeout. To first
order, the uncertainty on $Y_p$ scales as[39]:

$$\frac{\Delta Y_p}{Y_p} = 0.0390 \left( \frac{\Delta \Omega_b h^2}{\Omega_b h^2} \right) + 0.732 \left( \frac{\Delta \tau_n}{\tau_n} \right).$$

(1.21)

Various astrophysical measurements on stellar nuclei use spectroscopy of metal-poor
galaxies and the Cosmic Microwave Background for precise determinations of $Y_p$ and
$\Omega_b h^2$[40, 41]. These provide a relative uncertainty on $\Omega_b h^2$ of $\left( \frac{\Delta \Omega_b h^2}{\Omega_b h^2} \right) \sim 7 \times 10^{-3}$. At
present, the neutron lifetime is the largest source of uncertainty in the determination
of the relative abundance of primordial Helium. Following equation (1.21), this will no
longer be the case if $\frac{\Delta \tau_n}{\tau_n} < 4 \times 10^{-4}$. 13
2.1 INTRODUCTION

The lifetime of the free neutron has a long history, with recent work providing additional impetus for its study. Snell and Miller provided the first estimate of the neutron lifetime in 1948, 16 years after its discovery by Chadwick[42]. They initially proposed a lifetime of $\sim 30$ minutes, and additionally claimed that the lifetime was longer than 15 minutes. Two subsequent experiments, by Snell in 1950 and Robson in 1951, found lifetimes of $20 \pm 10$ minutes and $12.8 \pm 2.5$ minutes respectively. This began a long series of lifetime measurements, with a decreasing value of the lifetime as experiments improved precision. Historical measurements of the neutron lifetime can be seen in figure 2.1.

The neutron itself is difficult to detect. Many well-characterized particle detectors use some type of scintillation to convert an electric charge to light. The neutron has no electric charge and thus must be converted to some type of charged particle, typically through capture on a nucleus. As a result, many early neutron decay experiments instead utilized the charged products of the decays. Since the resultant $p^+$ and $e^-$ have a charge and thus scintillate, they are much easier to count.

This chapter briefly discusses previous measurements of neutron decays. Precision measurements of neutron $\beta$-decay have been studied through a wide suite of experiments. Prior to discussing experiments regarding the lifetime $\tau_n$, for completeness, a brief overview of asymmetry measurements is included in section 2.2. Measurements of the neutron lifetime typically use one of two different techniques: “Beam Experiments” and “Bottle Experiments”[43]. The earliest measurements of the neutron lifetime used the “beam method,” which will be summarized in section 2.3. Beam experiments, as
Figure 2.1: Historical measurements of the neutron lifetime. The black marks indicate the PDG values for the neutron lifetime. Present PDG values only use the bottle method. The bands show the presently accepted measurements of $\tau_n$ using either the beam or bottle method. Previous UCN$\tau$ measurements are noted separately from other bottle lifetime measurements, but are included in the bottle lifetime band.

The name suggests, utilize a beam of neutrons and detect the decay products. With the development of UCN sources, lifetime experiments involving the storage of low-energy neutrons have also been developed. The focus of this work, UCN$\tau$, is one example of a bottle experiment. Other bottles will be described in section 2.4. Finally, there are some neutron lifetime experiments that defy easy classification. Some brief discussion of these can be found in section 2.5.

2.2 EXPERIMENTAL MEASUREMENTS OF NEUTRON DECAY ASYMMETRIES

There are many observables from neutron decay besides the neutron lifetime. These angular correlation asymmetries, seen in equation (1.3), provide insights into the SM, in
particular the $V - A$ structure of charge currents. Many different angular decay correlation observables exist, relating the momentum or spin of the neutron decay products to each other. Each of these decay correlations provides an independent measurement of $\lambda = g_A / g_V$[6], allowing for the use of these observables to overconstrain $\lambda$. Lattice QCD calculations have been done to solve for $g_A$, but they have not reached the precision of experiment[44]. As a result, experimental measurements of neutron $\beta$-decay are used to constrain this parameter.

The $e - \bar{\nu}_e$ correlation parameter, $a$, is the angular correlation between the outgoing momenta of the three-momenta of the two resultant leptons in neutron decay. This relates to $\lambda$ through[45]:

\[
a = \frac{1 - \lambda^2}{1 + 3\lambda^2}.
\]  

Measurements of $a$ use the shape of the recoil $e^-$ spectrum to extrapolate the momentum of the $\bar{\nu}_e$. Using conservation of momentum, the momentum of the $\bar{\nu}_e$ can be extracted by measuring the $e^-$ and $p^+$ momenta accurately[46]. Precise measurements of $a$ measure the time-of-flight asymmetry for the produced electron and proton. Since the electron and proton are produced in coincidence, the momentum correlations can be readily extracted[47].

Another correlation coefficient is the $\beta$-decay asymmetry $A$, which measures the correlation between the neutron’s spin $\sigma_n$ and the electron’s momentum $p_e$. In terms of the free parameter $\lambda$, this is:

\[
A = -2\frac{\lambda(\lambda + 1)}{1 + 3\lambda^2}.
\]

Experimentally, polarized neutrons are used to extract the momentum direction of the produced electrons. The most recent precise results either use a beam of polarized neutrons or polarized UCN stored in a vessel[48, 49]. The resultant electron energies can be reconstructed in a spectrometer to determine the energy. Since the spin state of the neutrons is known precisely, this then allows the extraction of $A$. 
Related to $A$ is the antineutrino asymmetry $B$, which measures the difference between the neutron’s spin $\sigma_n$ and the antineutrino’s momentum $p_\nu$. Similarly to measurements of $a$ and $A$, the determination of $B$ precisely reconstructs the electron energy, while also counting the protons in coincidence[34]. $B$ can be written as a function of $\lambda$ through:

$$B = 2\frac{\lambda(\lambda - 1)}{1 + 3\lambda^2}. \quad (2.3)$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$-1.2756 \pm 0.0013$</td>
</tr>
<tr>
<td>$A$</td>
<td>$-0.11958 \pm 0.00021$</td>
</tr>
<tr>
<td>$B$</td>
<td>$0.9807 \pm 0.0030$</td>
</tr>
<tr>
<td>$a$</td>
<td>$-0.1059 \pm 0.0028$</td>
</tr>
</tbody>
</table>

Table 2.1: Particle Data Group (PDG) Values of neutron decay parameters as of 2020[14]

In conjunction with measurements of $\tau_n$, various measurements of the $\lambda$ parameter provide precision measurements of the SM. The present state of the asymmetry measurements can be seen in table 2.1. This work does not provide additional constraints on the decay correlations of the neutron. Nevertheless, the results of neutron lifetime measurements in conjunction with asymmetry experiments provide purely neutron based tests of the SM.

2.3 BEAM EXPERIMENTS

A beam experiment measures the decay products of a beam of neutrons. Like all radioactive decays, the number of neutrons can be fit to an exponential process. Given an initial number of particles $N_0$ with a characteristic lifetime $\tau_n$, the number $N$ remaining after some time $t$ will be:

$$N(t) = N_0 e^{-t/\tau_n}. \quad (2.4)$$

When measuring a lifetime based on the decay products, equation (2.4) can be rewrit-
ten as a differential equation on the decay rate. Both the neutron counting rate, $\dot{N}_n$ and decay product counting rate, $\dot{N}_p$, can be measured. Both of these rates have some measurement efficiency, $\epsilon_n$ and $\epsilon_p$ respectively. Using neutrons with velocity $v$ that decay inside of some known volume of length $L$, the lifetime can then be extracted as[50]:

$$\tau_n = \frac{L \dot{N}_n / \epsilon_n}{v \dot{N}_p / \epsilon_p}.$$ (2.5)

In order to measure the lifetime accurately, the knowledge of the efficiencies and knowledge of the rates must be known to a high degree. Recent beam experiments measure these quantities through different ways. The most accurate beam lifetime, at the National Institute of Standards and Technology (NIST) reactor neutron source, uses $^6$Li to detect neutrons and an electric Penning trap to store protons and guide them to an external proton detector[51]. An alternate beam lifetime experiment at the Japan Proton Accelerator Research Complex (J-PARC) instead utilizes a Time Projection Chamber filled with $^3$He. This then uses the capture reaction $^3$He + $n \rightarrow p + ^3$H, in conjunction with measured $\beta$-decay results[52, 53]. At present, no beam lifetime experiment has reached a precision below 1 s, but work is being done to achieve this.

### 2.4 BOTTLE EXPERIMENTS

A different class of lifetime experiments, bottle experiments, trap UCN inside a container for some period of time. The lifetime can be calculated from equation (2.4), measuring the resultant $N$ for a variety of holding times. Equation (2.4) can be rearranged to fit to:

$$\tau_{\text{meas}} = \frac{t}{\ln(N(t)/N_0)}.$$ (2.6)

This process must be repeated for multiple holding times. As the initial number of neutrons $N_0$ cannot be directly determined, the difficulty in measuring $\tau_n$ this way then comes from measuring the $N_0$ and $N(t)$ accurately. It should also be noted that the
measured lifetime, $\tau_{\text{meas}}$, could differ from the actual neutron lifetime. In the event of any extra sources of loss with a characteristic lifetime $\tau_{\text{loss}}$, the measured lifetime will be:

$$\tau_n^{-1} = \tau_{\text{meas}}^{-1} - \tau_{\text{loss}}^{-1}$$  \hspace{1cm} (2.7)

In order to accurately determine the lifetime, the $\tau_{\text{loss}}$ must be known sufficiently well. Material traps use Monte Carlo (MC) simulations, in conjunction with varying the vessel size, to determine the $\tau_{\text{loss}}$ from neutron interactions with the wall of the neutron container[54, 55]. After this, an extrapolation of $\sim 10$ to $100$ s is typically made to determine $\tau_n$.

In order to reduce the size of the extrapolation, magnetic traps avoid material losses by levitating neutrons in magnetic fields. Without directly interacting with walls, the $\tau_{\text{loss}}$ then will be dominated by other sources of loss such as depolarization or gas scattering. Magnetic traps contain neutrons with a magnetic field, which can be created with either permanent magnets or superconductors. These have been shown to have good control of systematic uncertainties, with comparable overall uncertainties to material bottles[56, 57].

### 2.5 OTHER EXPERIMENTS

Not all neutron lifetime measurements can be described using easy definitions like beam or bottle. Recent studies done with satellite instruments have also made preliminary determinations of $\tau_n$[58, 59]. These work by measuring incident neutrons generated by galactic cosmic rays interacting with the atmosphere or surface of celestial bodies like planets in our solar system. When these high energy cosmic rays strike the surface of a planet, they produce spallation neutrons. These can then be measured with various on-board detectors as probes fly by the object of interest. Given a precise knowledge of the atmospheric or surface conditions of the planet, the number of produced spallation
neutrons can be modeled. Over interplanetary distances, the actual neutron counting rates vary with the lifetime. The discrepancy between produced spallation neutrons and neutrons actually counted can then be used to extrapolate the lifetime. This provides a novel space-based approach to measure the neutron lifetime, complementary to terrestrial experiments. Unfortunately, this novel method of lifetime calculation has not yet reached the level of uncertainty of other experiments.

2.6 SUMMARY

Low energy neutron decay experiments can provide insight into the standard model in ways that cannot be probed with high-energy accelerators. The neutron lifetime, in conjunction with decay correlation coefficients, provides an independent measure of CKM matrix unitarity without the need for nuclear structure calculations. Additionally, the neutron lifetime acts as an input for various other physics processes.

Many measurements of the lifetime have already been performed. Among them, magnetic storage of neutrons as a direct measurement of the lifetime has the advantage of minimizing external loss mechanisms. This provides a very good understanding of potential systematic sources of loss. Next-generation high-precision measurements are being performed to push the overall relative uncertainty on the neutron lifetime down to $\mathcal{O}(10^{-4})$. 
CHAPTER 3
THE UCN\(\tau\) EXPERIMENT

3.1 INTRODUCTION

The UCN\(\tau\) experiment is a bottle-type experiment, designed to store Ultra Cold Neutrons (UCN) in a magnetogravitational trap for varying lengths of time. The use of magnets serves to minimize the loss mechanisms by eliminating interactions with the wall. Any source of loss besides neutron decay would lead to a shift in the measured lifetime.

This section describes the experimental setup of UCN\(\tau\). “Upstream” of the trap, described in section 3.2, includes regions which UCN pass through prior to entering the trap. Generation of spallation neutrons and reducing them to ultracold temperatures will be described in section 3.2.1. Polarization of neutrons for storage will be described in section 3.2.2. Transfer of UCN from the source to the trapping volume and the location of normalization monitors and Gate Valve (GV)s will also be discussed.

The “trap” itself refers to any parts of the UCN\(\tau\) apparatus which UCN might interact with after a filling period. The trapping volume, described in section 3.3.1, provides the actual magnetogravitational mechanism for storage. Various moving parts allow for the UCN\(\tau\) trap to measure neutrons in situ. The Trap Door (TD) and Cat Door (CD), in section 3.3.2, the dagger, in section 3.3.3, and the cleaner, in section 3.3.4, all move inside the vacuum chamber to manipulate or count trapped neutrons. Finally, the software controls and data acquisition systems, required to read out measurements of various aspects of the experiment, shall be described in section 3.4.

Figure 3.1 illustrates the experimental setup, including the UCN\(\tau\) guide system and the UCN\(\tau\) trap during the 2018 run cycle. In 2017, the Roundhouse (RH) had not been
Figure 3.1: Annotated schematic of the 2018 UCNτ trap system. UCN enter the guides from the left, at the arrow labeled “Source”. Moving parts of the experimental setup, including two gate valves, are labeled with red triangles. The two magnets, the PPM and AFP, are indicated with blue rectangles. Monitor detectors are indicated with green ovals. In 2017, the RH was replaced by a straight guide.

installed and thus a straight guide passed between the Pre-Polarizing Magnet (PPM) and Adiabatic Fast Passage (AFP) magnets. The location of the moving elements, excepting the Roundhouse Gate Valve (RHGV), are the same between 2017 and 2018.

3.2 UPSTREAM OF TRAP

3.2.1 Source

UCN are produced at the Los Alamos Neutron Science Center (LANSCE) Neutron Source, which consists of a tungsten spallation target bombarded with 800 MeV protons[60]. The proton beam arrives in bursts of between 5 and 10 proton pulses, with an accelerator rate of 20 Hz and a interarrival time between bursts of 5 s. During each run, this ultimately gives a \( \sim 9 \mu A \) average current on the target.

The spallation neutrons are first moderated and reflected by room-temperature beryl-
lium and graphite. These neutrons are further moderated by polyethylene cooled to \( \sim 45 \) K by helium gas boil-off. After this moderation, the resultant cold neutrons become UCN through phonon interactions in a solid D\(_2\) crystal cooled by liquid helium to 5 K. As para-D\(_2\) upscatters UCN, the D\(_2\) in the source has been converted predominately to the ortho-D\(_2\) spin state, with the fraction of para-D\(_2\) at \( \sim 3 \% \)[60].

The source has a “flapper valve” open only while the beam is on, opening for a duration of 1 s every 5 s. The flapper increases UCN output by keeping them away from potential absorption by the D\(_2\). UCN leave the source through a 1 m vertical \(^{58}\)Ni coated guide, which has an \( E_f = 335 \) neV[13]. They are guided into the experimental area via a horizontal NiP guide, which has an \( E_f = 213 \) neV[61]. Multiple guides exit the shielding stack which contains the neutron source, each guide with their own independent Gate Valve (GV). In this way, multiple experiments can be run simultaneously.

3.2.2 Magnets

In order to store neutrons in a magnetic field, they must be polarized to the “low field seeking” spin state. UCN generated by the source have no intrinsic polarization state. Since UCN\(\tau\) and other experiments using UCN at Los Alamos National Laboratory (LANL) such as UCNA and nEDM utilize magnetic properties of UCN, various magnets are used to polarize UCN leaving the source before being directed into the experiments.

The Pre-Polarizing Magnet (PPM) is a 6 T superconducting magnet immediately after the GV between the source and the guides. Low field seeking UCN do not have enough energy to pass through this magnetic potential barrier. This strong magnetic field thus polarizes all trappable neutrons entering the experimental area, making them “high field seeking.” A thin foil inside the guide separates the source from the experimental vacuum.

As actual trapping of neutrons requires low field seeking neutrons, UCN after the PPM pass through an Adiabatic Fast Passage (AFP) spin flipper. This is a combination
of a static magnetic field, $B_0 \sim 14 \text{ mT}$, with a radio frequency component $\omega$ perpendicular to $B_0$. As polarized UCN pass through the field, they will adiabatically change from the “high field seeking” state to the “low field seeking” state, as the perpendicular component of the field rotates by $\pi$ radians. In the beginning of each run cycle, the components of this field are tuned by scanning $B_0$ and $\omega$, in order to find the resonant frequency for UCN passing through. During this process, the UCN\text{\textgreek{t}} trap acts as a spin analyzer, as the strong magnetic fields capture high field seeking UCN by pulling them towards the walls of the trap, while repelling low field seeking UCN to trap them.

3.2.3 Guides and Roundhouse

The space between the source and the trap attempts to condition and maximize the number of trappable UCN inside the UCN\text{\textgreek{t}} apparatus. The “guide” system consists of 3 inch or 3.125 inch outer diameter pipes coated with high $E_f$ materials. After exiting the source, UCN pass through a primary GV. After this GV, a horizontal NiP guide, which has an $E_f = 213 \text{ neV}$, runs through the PPM. A nonmagnetic quartz guide coated with diamond-like carbon (DLC), with $E_f = 249 \text{ neV}$, runs through the AFP magnet[62]. The transition region between the guides and the trap has Cu plating, with an $E_f = 168 \text{ neV}$.

In between the 2017 and 2018 run cycles, the guide system was upgraded with the introduction of a buffer volume. The Roundhouse (RH) is a stainless steel vessel coated with NiP, present for almost all data taking in 2018. This volume preconditions the energy spectrum of UCN and reduces the impact of current fluctuation of the proton beam pulses. In order to avoid depolarization, the base plate of the RH is Aluminum coated with NiP. Additionally, the RH has been wrapped by coils providing a tuneable magnetic field to avoid field zeros and the resulting depolarization. The RH connects to the guide system via two GVs, one upstream and one downstream. For normal running these two GVs have been synced together so that they open and close at the same time. Additionally, a polyethylene neutron absorber was placed on the downstream side of
the downstream GV to remove UCN trapped in the guides between the trap and the RH during running.

3.2.4 Monitoring Detectors

Not all neutrons traveling from the source end up in the UCN$\tau$ trap. Along the path of the guide are various monitor detectors, which each sample different regions of the energy distribution. These monitor counts become important in reconstructing the number of trapped neutrons, as the prerequisite of measuring the neutron lifetime is to account for fluctuations in the source production.

Three monitor detectors are present in both the 2017 and 2018 run cycles. The first of these is the GV monitor, located in a pinhole in the guide between the source and the north Gate Valve that allows neutrons to enter the experiment. Above the trapping height of the UCN$\tau$ trap, the Standpipe (SP) monitor counts overthreshold neutrons after spin flipping and immediately prior to their entry into the trap. Behind the Trap Door (TD) and Cat Door (CD), the Downstream (DS) monitor measures neutrons remaining in the guide system, and also measures some small amount of leakage UCN during the fill.

With the introduction of the RH in 2018, two monitors are incorporated into its structure. At the bottom of the RH is the RH monitor, which samples neutrons of all energies inside the RH. At the top of the RH, the Roundhouse Active Cleaner (RHAC) monitor is a Photomultiplier Tube (PMT) that detects scintillation light produced by neutrons hitting the adjustable cleaner at the top of the RH. These two monitors replaced the “Foil” and “Bare” monitors, which were located in a similar position to the GV monitor. To provide spectral information, the Bare and Foil monitors differed from the other pre-GV monitors by the size of their pinholes, and the presence of a thin Aluminum foil to block low-energy UCN.
3.3 TRAP

Figure 3.2: Cutaway model of the inside of the UCNτ trap. The magnetic fields used to trap UCN are the Halbach array and the holding field coils. Not all of the holding field coils have been marked; many have been removed to see inside the trap. In this model, the dagger detector has been lowered to the bottom of the trap. The two cleaners have been marked as well. In this view, the left cleaner is the “giant cleaner” while the right cleaner is the “active cleaner.”

3.3.1 Halbach Array

Neutrons for the UCNτ experiment are stored in a magnetogravitational container, seen in figure 3.2. A Halbach array of permanent magnets forms a “bathtub” shaped storage volume. Approximately 4000 Neodymium permanent magnets, each coated with Aluminum, form the lower surface of the trap. Each magnet has dimensions of
2.54 cm × 5.08 cm × 1.27 cm, with a remnant magnetic field of 1.35 T. The Halbach array configuration rotates the magnetization vector of adjacent magnets by 90°. The magnets lie along the intersection of two tori described with radii 1 m and 0.5 m. The two tori have the same radii with the major and minor axes flipped. Additionally, these tori only reach a total height of 0.5 m, leaving the top of the trap open while gravity effectively forms the top lid of the trap. This serves to create an asymmetric trap with a volume of ∼420 L.

Only low-field seeking UCN can be trapped in the array. In the absence of an external field, UCN can depolarize and thus spontaneously flip to the high-field seeking state. To ensure that UCN do not pass through a field zero, the vacuum vessel surrounding this magnet array is wrapped by 10 “holding field” doorframe electromagnets. A power supply provides current through the holding field coils. This power supply provides a maximum current of ∼300 A, corresponding to a magnetic field of ∼10 mT. The resultant magnetic field can be analytically modeled; for explicit forms of the fields inside the trap see section 5.2.

3.3.2 Trap Door

The UCN̂ trap sits elevated above the guide system. In order to move UCN from the guides to the inside of the trap, the “Trap Door” (TD) vertically raises to bring neutrons into the trap. The TD consists of a long piston shaft, driven by a large gear coupled to a servo motor. The trap has a 6 × 6 in² hole at the bottom center of the Halbach array; a matching 6 × 6 in² Halbach array sits atop the TD shaft. As the servo motor drives the gear, the piston moves up and plugs into position to close the bottom of the trap magnet array. Reversing the direction of the motor lowers the piston and opens the trap to the guide system. The piston is put under vacuum with a kinematic seal using two O-rings; the teflon-coated O-rings wear under the piston motion and must be replaced after approximately 1000 TD cycles.
The piston and array travel through a transition region between the guides and the trap. This region contains an additional “Cat Door,” which sits between the AFP and the trap. The CD is a rectangular copper plate that rotates via a stepper motor. During normal running, the CD moves in conjunction with the TD to change the volume accessible to UCN. When the TD moves or stands in the “up” position, the CD stands vertically, blocking UCN from leaving the guides. During filling, the CD sits at a 45° angle so that UCN can be directed upwards to enter the trap. After the run, the CD lies flat so that UCN exit the trap to be counted by the downstream detector.

The Trap Door and Cat Door are driven by 5 V logic into a Parker Compax3S drive. For the 2017 and 2018 running, a Python server sent these logical voltages to the Compax3S drive over USB. This drive simultaneously controlled the motors for both the TD and CD motors, with error catching logics so that the two moving parts could not run into each other.

3.3.3 Dagger Detector

The principal neutron detector for the UCNτ experiment, referred to as the “dagger” detector, counts neutrons in-situ. The dagger is a 40 × 20 × 0.6 cm³ prism, with the lower, ~20 cm, edge cut conform to the curvature of the Halbach array. It is attached to a linear actuator, allowing it to raise up to 49 cm above the bottom of the trap and lower it to the bottom of the trap. This allows the dagger to lower into the trap to count UCN remaining in the storage volume. Limit switches prevent the dagger from traveling above 45 cm or below 1 cm, measured from the lowest point of the trap. Driving the dagger into the top of the vacuum vessel or the bottom of the trap could cause the dagger to decouple, damaging both the detector and the array.

The detector uses an Elgen manufactured “EJ-442” Zinc Sulfide (ZnS:Ag) alpha detector, coated with a layer of 95% enriched ¹⁰B[63]. This multilayer neutron detector captures UCN and produces scintillation light. Photons from the ZnS:Ag scintillator are
guided to one of two PMTs by an array of wavelength-shifting fibers. The fibers alternate so that each UCN produced light event appears in both PMTs simultaneously. This captures UCN; the light collection efficiency is estimated at $0.961 \pm 0.003^{[64]}$. Scintillators are attached to both sides of the detector, so that the insertion of the detector does not reflect UCN and significantly change the phase space distribution, other than the edges of the detector housing.

In between the 2017 and 2018 running cycles, the dagger detector was replaced due to damage along its curved edge. This replacement additionally doubled the $^{10}\text{B}$ layer thickness, roughly doubling the efficiency of the detector.

3.3.4 Spectral Cleaners

High energy UCN enter the trap from the source, but might not be trapped by the Halbach array. In order to remove overthreshold UCN so that they don’t escape during storage and bias the lifetime measurement, the trap contains two “cleaners,” which lower into the trap to remove high energy UCN. During filling and a dedicated cleaning period prior to the hold, these two cleaners sit at 38 cm above the bottom of the trap. During the hold, these two both retract to a height of 43 cm above the bottom of the trap. This serves to minimize the risk of UCN finding the cleaning surface during the holding time.

The smaller of the two cleaners, the “Active Cleaner (AC),” has a rectangular surface of $66 \times 36$ cm$^2$. The AC surface utilizes the same technology as the dagger detector; it uses $^{10}\text{B}$ coated ZnS:Ag in conjunction with PMTs to detect overthreshold neutrons that then are removed from the trap. This detector provides useful information for both normalization and systematic studies. The other cleaner is typically referred to as the “Giant Cleaner (GC).” The GC shape is custom fit cover 1/2 of the cross section of the trap at the cleaning height of 38 cm. It utilizes a solid sheet of polyethylene to remove high-energy UCN. Polyethylene has a very high upscattering cross-section for UCN and a nearly zero potential barrier, and thus provides a very efficient means of removing.
overthreshold neutrons. The AC was driven through a pneumatic cylinder. In 2017 the GC used a Nook Industries CC Series Compact Cylinder screw drive, allowing for more precise position tuning. Electric noise from this drive led to significant cross-talk with the dagger detector. As a result, in 2018, the GC actuator was replaced by a pneumatic cylinder.

One studied method of “activating” the giant cleaner utilized solid wavelength-shifting sheets, with Silicon Photomultiplier (SiPM) arrays mounted along the sides of the detector. These Elgen manufactured “EJ-280-10x Green WLS Plastic“ wavelength shifting sheets were cut to be $50 \times 30 \times 0.5 \text{ cm}^3$, attached to mounted $^{10}\text{B}$-coated ZnS:Ag neutron detector sheets. Up to 8 photon detectors were coupled via optical epoxy to each wavelength shifting sheet. These detectors consisted of four SensL “MicroFC-60035-SMT-A1” SiPMs connected in series with an amplifying circuit. Raw signals from the photomultipliers were boosted through a transimpedance amplifier formed by a capacitor and a Mini-Circuits “TC9-1-75+” transformer. These arrays were powered through a coaxial cable, attached to a printed circuit board via Hirose Connectors brand “798-U.FL-R-SMT-110” coaxial connector. Custom “FMCA-1024-60” cables, manufactured by Fairview Microwave, propagated signals from the detectors through the vacuum feedthrough.

A Geant4 simulation characterized the response of the giant cleaner detector[65]. The results of such a simulation can be seen in figure 3.3. Light from UCN incident upon the detector decays away with an attenuation length of $11.0 \pm 1.9 \text{ cm}$. Much of the light from UCN interactions does not reach the detector arrays; the solid angle of light produced by each interaction with a direct line to the array can be very small. Given multiple SiPM arrays, this geometric constraint could be used to reconstruct the location of overthreshold UCN. Unfortunately, preliminary results from this detector as installed in UCN showed significant amounts of excess noise. As a result, the position dependence has not been adequately reconstructed.
Figure 3.3: Simulated detector response for UCN on a sheet of wavelength-shifting plastic for the active giant cleaner. Neutrons have been generated with a uniform distribution across the surface, with an attached detector along the $(\hat{x}, \hat{z})$ plane corresponding to one SiPM detector $-250 \leq x \leq -246$ mm. The attenuation length of light in the detector has been determined to be $11.0 \pm 1.9$ cm, with additional dark regions due to geometric constraints.

3.4 SOFTWARE CONTROLS AND DATA ACQUISITION

3.4.1 Data Acquisition

A single Data Acquisition (DAQ) computer both controls the motion of trap components and records data from the many monitors into a readable format. A dedicated control Graphical User Interface (GUI) sends control signals to power air driven solenoids to open GVs and move actuators, such as the cleaner. For most of the 2017 and 2018 running cycles, these control signals fed into a LabJack which managed the actuation. In order to reduce potential timing jitters, later runs used an Arduino to drive these valves. Additionally, the trap door and dagger actuators had dedicated Python servers which
drove movement at specified times through the computer. The control GUI was capable of loading preset run configurations, with pre-specified timings, in order to maintain consistent timings during a run.

An additional GUI on the same computer drove the data acquisition. Data from 10 monitor channels is read into two Fast Comtec MCS6A digitizers. Additionally, each digitizer featured 5 possible “tagbits,” recording motion signals from the control output. For more about reading tagbits, and applications to analysis, see section 4.3.2. These two digitizers are synced at the beginning of the run and can resolve events at the level of 800 ps. The DAQ GUI reads in raw binary data from the digitizers. This raw data is then blinded and saved in two formats: blinded binary and ROOT trees[66]. The blinding algorithm used can be found in section 4.1.3.

For the analysis described in subsequent chapters, the binary format will be ignored in favor of ROOT trees. These trees, with one representing each of the two Multi-channel Analyzer (MCS) digitizers, are then parsed by a suite of custom C++ codes to extract summed observables into Comma Separated Variable (CSV) files. These C++ codes have been designed to run in parallel on the Indiana University (IU) Carbonate supercomputing cluster so that extracting summed values from each tree takes ~30 minutes for the entire dataset. The extracted CSV files can then be read by Python codes for analysis of the neutron lifetime and associated systematic effects.

3.4.2 EMS and RINGTAIL

The Environmental Monitoring System (EMS) consists of an SQL database using a number of Python scripts, each reading devices in the experimental area. In particular, this allows monitoring pressures and temperature inside various parts of the experiment and source. The EMS also reads data from the LANSCE beamline, allowing real-time monitoring of magnet failures and proton source arcdowns.

Some values from the EMS are incorporated into the UCN$\tau$ output ROOT files: trap
pressures and PMT temperatures. However, the EMS does not have the same time precision as the MCS, and thus we cannot use any of these values for more than a gross analysis for quality checks. With higher precision it might be possible to relate source temperature with monitor counts, but at present aliasing on the order of 10 seconds prevents this analysis.

To automate the run-sequence control and data-taking, and prepare for reduced manpower during overnight running, the Run Initiating Neutron Gathering Tau Assistant Independent Logger (RINGTAIL) was introduced after the 2018 run cycle. This is an additional Python script that parses recent entries in the SQL database and sends text message alerts if EMS parameters stray outside a preset range. Watchdog parameters can be added to this range by modifying the setup .csv file. The DAQ computer can be accessed, via the NoMachine software, by other computers at remote institutions, such as IU, to remotely start and stop runs. In the event of major system failures, someone at LANL has to physically control things. However, remote access allows off-site experimenters to change run configurations and control most components of the experiment remotely.
CHAPTER 4
LIFETIME ANALYSIS

4.1 INTRODUCTION AND OVERVIEW OF ANALYSIS

4.1.1 Overview

The UCN\(\tau\) lifetime result comes from 3 independent analyses. Described here is one of those for two data sets, consisting of data taken during the calendar years 2017 and 2018. The analysis methods here are broadly applicable to previous and future sets of data, as many “bottle” measurements of the neutron lifetime have similar analytic and systematic difficulties.

The \(\beta\)-decay decay of neutrons follows an exponential decay function, where the number \(N(t')\) of neutrons at a given time \(t'\) with a lifetime \(\tau_n\) can be written as:

\[
N(t') = N(t'_0 = 0) e^{-t'/\tau_n}.
\]  

As a bottle type experiment, UCN\(\tau\) measures the number of Ultra Cold Neutrons (UCN), \(N\), remaining in the trap after some amount of time. More accurately, UCN\(\tau\) measures a set of observable counts that must then be reconstructed to determine \(N\) and extract \(\tau_n\). Each “run,” denoted with the index \(k\), consists of a single measurement, with a known “holding time,” \(t_k\), a number of upstream “monitor” responses, \(M_{j,k}\), and integrated “unload” counts from the dagger detector \(U_k\). First, each run must convert raw “unload,” \(U_k\) counts into a corrected “yield,” \(Y_k\). This proceeds by adding in a “rate-dependent correction,” \(\epsilon_{U_k}\), and a “background correction,” \(B_k\), giving:

\[
Y_k = U_k + \epsilon_{U_k} - B_k.
\]
Subsequent sections deal with the determination of these parameters. More information on unloads, $U$, and rate dependent effects, $e_{U}$, will be found in section 4.4. Background models will be discussed in section 4.5. Additionally, the number of neutrons filled in the trap at $t'_{0} = 0$ is unknown and could vary as the UCN source deteriorates. As a result, UCN$_{\tau}$ must utilize the upstream monitors to “normalize” the yields so that each can be compared. A number of UCN at $t'_{0} = 0$, $N_{\tau k}$, can be predicted based on the monitor counts of each run. This serves to “normalize,” based on the responses of other UCN detectors, the expected number of UCN in the UCN$_{\tau}$ trap. Normalization procedures will be discussed in section 4.6.

The lifetime $\tau_{n}$ can then be extracted from a measurement of at least two runs. Given a pair of runs, one with a short holding time, $t_{S} < 1000$ s, and one with a long holding time, $t_{L} \geq 1000$ s, each with their associated yields from equation (4.2) and expected counts $N_{\tau S}$ and $N_{\tau L}$, the lifetime can be determined directly:

$$\tau_{n} = \frac{t_{L} - t_{S}}{\ln \left( \frac{Y_{S} N_{\tau L}}{Y_{L} N_{\tau S}} \right)}.$$ 

Combining a large number of repeated runs, the lifetime can be calculated through an optimization procedure to determine the best values of all these input parameters. In the event that UCN$_{\tau}$ has additional loss mechanisms besides neutron decay, equation (4.3) must be modified to incorporate these systematic shifts. Systematic losses of UCN during the holding periods will be described in detail in chapter 6, and for the most part will not be discussed in this chapter. Statistical precision limits the understanding of our observable input parameters, and thus contributes to the primary source of uncertainty in UCN$_{\tau}$.
4.1.2 Production Running

The data for the neutron lifetime calculation comes from a set “production” data runtype. A production run requires neutrons to enter the trap, be held for some amount of time, and then be counted immediately after the storage.

![PMT Counts, Run 4230](image)

Figure 4.1: Typical raw dagger detector output data for one production run. The red histogram plots “singles” events triggering on Photomultiplier Tube (PMT)1, the blue histogram plots the “singles” events triggering on PMT2, and the green histogram plots “coincidence” events. The coincidence counting provides a factor of $O(100)$ improvement for signal-to-noise. Vertical black lines indicate motion within the trap; the fill ends at 150 s, and the holding time begins at 200 s and ends at 220 s. The trap door moves at 150 s, and then again between 430 s and 440 s. The three dagger unload steps at 220 s, 260 s, and 280 s, correspond to heights of 38 cm, 25 cm, and 1 cm respectively.

An example production run can be seen in figure 4.1. The “fill” cycle opens the Trap Door (TD) and all gate valves between the source and the UCN$\tau$ apparatus. The Cat Door (CD) sits at 45°, allowing UCN to move between the upstream guides and the trap. The proton beam hits the target and produces UCN, which flow into the trap. The
dagger height is set to 38 cm above the bottom of the trap, and both cleaners are lowered to the cleaning height of 38 cm. Reaching saturation requires 150 s for a straight guide between the source and trap (in 2017), and 300 s with the roundhouse installed (in 2018).

At the end of filling, the beam is turned off. During the “cleaning” cycle, gate valves between the source and the trap, as well as the trap door, are all closed. The dagger moves to 49 cm above the bottom of the trap, while the cleaners stay down to remove overthreshold neutrons.

The “hold” period lasts for a programmed variable length of time, with the cleaners, dagger, and trap door all up. During production running, the clock timing of Multi-channel Analyzer (MCS) timestamps shifts during the hold by a constant factor, unknown to the analyzer. This blinding scheme gives a possible shift in the lifetime of ±15 s[56]. All holding times cited below incorporate this blinding factor. This analysis divides runs into two sets, relating to holding times: “Long” runs have holding times $t \geq 1280$ s, while “short” runs have holding times $t \leq 200$ s.

After the hold, the dagger lowers during the “unload” period. For most production data in 2017 and 2018, the dagger moves in three “dips”, staying for 40 s at 38 cm (our cleaning height), then 20 s at 25 cm and 150 s at 1 cm, slightly above the bottom of the trap. Additionally, in 2018 the active cleaner lowers during this time as well, to measure the overthreshold UCN.

After the unload, the trap door opens up again, while the cat door lies flat in the “dump” position, allowing access to the guides and a downstream monitor. During this time, a background measurement is taken with the dagger at the bottom of the trap for 50 s. These production run timings are summarized in table 4.1. Other non-production runs were also taken, predominately to investigate potential systematic issues with the experiment. Unless otherwise noted, this chapter deals entirely with production running.

Two calendar years worth of data, containing 5590 individual “production” type runs,
Table 4.1: Typical times for different parts of the UCN\(\tau\) production cycle. Note that for this analysis the hold time is blinded.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill</td>
<td>150 (2017), 300 (2018)</td>
</tr>
<tr>
<td>Clean</td>
<td>50</td>
</tr>
<tr>
<td>Hold</td>
<td>20,50,100,200,1550,3000,4000,5000</td>
</tr>
<tr>
<td>Unload</td>
<td>210</td>
</tr>
<tr>
<td>Background</td>
<td>50</td>
</tr>
</tbody>
</table>

were used in this analysis. The two years vary primarily due to the presence of the Roundhouse (RH) buffer volume between the source and the trap, which is present for all production data from 2018. Additionally, between the two years, the dagger detector was completely replaced. The new thickness of the \(^{10}\text{B}\) layer in the 2017 dagger, \(\sim 10 \ \mu\text{m}\), roughly doubles in the 2018 dagger to \(\sim 20 \ \mu\text{m}\). As a result, UCN are counted faster in 2018, and backgrounds can be suppressed further than in 2017.

4.1.3 Blinding

The UCN\(\tau\) experiment blinds incoming data to avoid analyzer bias. The blinding process multiplies the actual holding time by a hidden factor, \(f\), encrypted on the Slow Control software, and shifts times recorded by the Data Acquisition (DAQ) accordingly. This scales the lifetime by \(f\), since for a given short/long lifetime pair with holding times \(t_S, t_L\) and yield \(Y_S, Y_L\):

\[
\tau_{\text{blind}} = \frac{ft_L - ft_S}{\ln(Y_L/Y_S)} = f\tau_n. \quad (4.4)
\]

An encrypted file on the DAQ computer contains \(f\), which is randomly generated between \((1 - 1.7 \times 10^{-2}) \leq f \leq (1 + 1.7 \times 10^{-2})\). This corresponds to a lifetime shift of \(\Delta(\tau_n) \sim \pm 15\) s.

The MCS records the raw time of each event as an unsigned long variable, counting “clock ticks” from the beginning of the run, where one clock tick corresponds to 0.8 ns of
real time. Each run control file contains two “Hold” flags that indicate the start and end of the blinded holding period. For every event with a raw time between these two flags, each clock tick is multiplied by the factor $f$. Individual timestamps are blinded through Algorithm 1, and these timestamps can be unblinded by reversing the algorithm.

**Algorithm 1** Blinding Algorithm

1: $0.983 \leq f \leq 1.017$
2: $t_r =$ MCS (real) time
3: $t_b =$ Blinded (output) time
4: $t_0 =$ End of Cleaning Period, Start of Holding Period
5: $t_f =$ Start of Counting Period, End of Holding Period
6: $t_h = t_f - t_0,$ Length of Hold
7: if $t_r < t_0$ then
8: $t_b = t_r$
9: else if $t_0 \leq t_r < t_f$ then
10: $t_b = t_0 + \frac{t_r - t_0}{t_f - t_0} t_h$
11: else if $t_{raw} \geq t_f$ then
12: $t_b = t_r + (f - 1)(t_f - t_0)$
13: end if

Only timestamps during the holding period are blinded. This means that PMT rates measured during the unload or filling periods are not scaled, but any PMT rates measured during the holding time will be scaled by $1/f$. As a result, background measurements should not be taken during the holding times, as these rates will be systematically limited to 1.7% precision. Background measurements therefore must come from a dedicated background period at the end of the run.

### 4.2 MONITOR COUNTS

#### 4.2.1 Overview

Each time UCN$\tau$ fills with neutrons, the source conditions directly impact the number of trappable UCN. Various monitor detectors, the position of which can be seen in figure 3.1, must be used to reconstruct the number of neutrons present in the trap prior to the unload. At any given time, up to five normalization monitors sample the
spectrum, each sampling a different population of UCN. For both years the Gate Valve (GV), Standpipe (SP), and Downstream (DS) monitors are present. In 2017, the Foil and Bare monitors are also available, while in 2018 there are two monitors in the RH. These monitors correspond to the overall source output, as each monitor essentially counts a constant, albeit unknown, fraction of the total UCN produced.

![Gate Valve Summed Counts](image)

Figure 4.2: Summed counts in the GV monitor during filling (150 s) in 2017. The GV monitor is closest to the source, and thus is directly proportional to the source output. The “sawtooth” behavior of this plot corresponds to source conditioning during running. Adding a new layer of Deuterium on the source increases the production, while the amount of UCN produced are gradually reduced as the source is hit by the proton beam. One of these “sawtooth” patterns corresponds to roughly 2 days of real time. Occasional source issues can also cause certain runs to have near-zero monitor counts.

The spallation source is not perfect; beam drifts, pulse drops, and source geometry affect the total number of UCN produced. In figure 4.2, the “sawtooth” behavior of UCN production stems from periodic changes due to the source Melt and Refreeze (MRF) cycle. Every two calendar days, approximately every 50 runs, the source must be...
recycled with a new deuterium crystal, leading to a discontinuity as the source production increases. However, proton beam current hitting the source can locally change the D$_2$ crystal conditions, potentially hardening the spectrum. As a result, the overall UCN production decreases on a run-by-run basis and the individual monitor ratio changes slightly. As UCN$\tau$ samples a specific subset of the overall UCN spectrum, the monitor counts provide an external indication of overall source performance and a means by which to predict UCN$\tau$ counts. More information on the predictive power of monitor counts will be described in section 4.6.

4.2.2 Monitor Count Structures

![UCN Rates Before/After RH, Run 11238](image)

Figure 4.3: Raw GV and RH monitor counts for one run in 2018. The two fills have been fitted to equation (4.6), with fixed time constants but free pulse amplitudes. The rate in the GV monitor has additional scatter due to the individual beam “bunches,” but these bunches are not present in the RH monitor. The RH monitor takes noticeably more time to reach saturation than the GV monitor.
Protons coming from the Los Alamos Neutron Science Center (LANSCE) accelerator produce UCN, which propagate through the guide system with characteristic timing and responses. The beam structure of incident protons consists of “bunches” of protons arriving on the spallation target every 5 s. Because of the bunched nature of the arriving protons, individual monitors see peaks corresponding to the accelerator current. These peaks can be seen in the GV fit curve in figure 4.3, which histograms the GV monitor and the RH monitor for the same run. The RH monitor reduces the scatter during the fill caused by beam drops and current variations among individual pulses. For a given volume, with a single saturation time constant, $\kappa$, and an expected total number of neutrons, $N_{\text{tot}}$, the number of trapped neutrons at any given time can be written as a single exponential:

$$N = N_{\text{tot}} \left( 1 - e^{-t'/\kappa} \right). \quad (4.5)$$

With a series of pulses, the neutron monitor rates can be described using a filling and draining time for each individual pulse. Each individual pulse, $\phi_i$, is allowed to vary in magnitude, multiplied by an offset rising exponential, $\kappa$, and a falling exponential, $\eta$. Each of these time constants should be the same for each pulse in a given monitor count, and thus the rate in a given monitor can be written as a sum:

$$\Phi(t') = \sum_i \phi_i \left( 1 - e^{-\frac{(t'-\tau_i)}{\kappa}} \right) e^{-\frac{(t'-\tau_i)}{\eta}}. \quad (4.6)$$

Fitting the pulse heights and timing structures of equation (4.6) can provide insight onto the monitor counts. To this end, ROOT’s histogram fitting algorithm fit the pulse heights and time constants for each monitor on each run, binned into 0.5 s bins for the histogram[66]. However, since the filling time lasts 150 s or 300 s, an individual fill contains 30 or 60 pulses, in addition to the time constants. Fitting a 63-dimensional function for each run leads to a significant amount of uncertainty, and thus the scatter in
pulses does not provide enough information to reliably distinguish good and bad runs, or provide a sufficiently clean estimate of weighted neutron filling. Even fixing the time constants $\eta$ and $\kappa$ does not improve the fit significantly. As a result, equation (4.6) should only be used for determining broad changes in the monitor responses.

Figure 4.4: Raw monitor counts during two typical short runs. Two monitors, the GV and SP are plotted for 2017, which has a filling time of 150 s. For 2018, the RH and Roundhouse Active Cleaner (RHAC) monitors are plotted instead, with a filling time of 300 s. After the filling time, neutrons either decay or are lost due to interactions with the volume. Each year has a “low” monitor which counts most of the UCN coming out of the source, and a “high” monitor, counting only higher energy UCN. In 2017, the low monitor is the GV and the high monitor is the SP. In 2018, the low monitor is the RH and the high monitor is the RHAC.

In 2018, the inclusion of the RH buffer volume helps to smooth out the waveform, lessening the impact of a few individual bad pulses. A comparison between the monitors used in 2017 and 2018 can be seen in figure 4.4. Both the GV and RH monitors sample neutrons from the bottom of the guide system, and thus select UCN with the full energy
spectrum. The SP and RHAC monitors count neutrons from above the top of the UCN\(\tau\) cleaning height, sampling the same region of energy space over the threshold of the trapping potential. Because UCN remain in the RH for a longer period of time than just in the guides, minor beam inconsistencies, particularly between individual proton pulses, are less significant, resulting in a smoother curve for the RH monitor data and more robust normalization.

4.2.3 Monitor Backgrounds

Normalization monitors have a non-negligible background that must be accounted for when calculating monitor responses. Raw counts vary on a run-by-run basis, and a poor estimate of the number of counts in monitors could potentially bias normalization efforts.

A typical production run has dedicated time at the end for background counting. Utilizing this time period for monitor counts, the background rates for the monitor counts can be calibrated. As can be seen in figure 4.5, short runs have remaining UCN that could bias the monitor readings. Since UCN could remain in the source or guide system for a very long period of time, the \(\sim 600\) s short holding time runs do not fully equilibrate with background. Additionally, during some runs the beam could be turned on during the holding time in order to run different UCN experiments at LANSCE. Occasionally the beam would mistakenly be turned on during the unload, which would cause elevated background rates in the dagger. In order to maintain constant run conditions, any runs where a GV or SP end of run rate was above 1\% of the rate during the fill were removed from the analysis.

In order to adequately describe the monitor counts, backgrounds must be taken from a combination of daytime dedicated backgrounds and the end of long holding time runs. Within each period with consistent monitor behavior, a constant monitor background rate is calculated and subtracted from the recorded monitor counts.
Figure 4.5: Comparison of rates in the GV between the filling time and the final 50 s of the run. Shorter holding times have elevated counts compared to longer holding times, indicating UCN remaining in the source for $O(100)$ s. Some runs have rates comparable at the end of run to the filling time. These occur due to either low rates during filling or due to the beam turning on during the unload. These outliers should be removed from analysis.

4.2.4 Monitor Weighting

The initial number of UCN in each monitor during filling does not need to be determined precisely. Instead, the important value from monitor reconstruction should be some singular value that provides the most reliable estimate of the UCN contribution in UCN$_{\tau}$ from each monitor. The monitor counts provide a spectral response value, which should be able to reproduce the behavior of filling the UCN$_{\tau}$ trap. Importantly, the function chosen should allow extra weighting towards neutrons entering the trap near the end of the fill, since these have the most import in establishing the trap’s UCN density.
As previously discussed in section 4.2.2, the number of neutrons filling a volume, equation (4.5), can be fit to a single exponential. The filling time constant, $\kappa$, can be fit on a run-by-run basis for each monitor in the trap. As individual monitor saturation time constants vary, the individually fit values for $\kappa$ might not provide enough information for reconstruction of the initial counts, $N_\tau$, in the trap. In particular, the saturation time of the RH is significantly longer. Since UCN$\tau$ monitors must reconstruct $N_\tau$ in the trap, the saturation time in the trap is the relevant parameter, which can be measured through the two dagger monitors. This exponential weighting time constant is chosen to be $\kappa = 70$ s for this analysis. Since the actual geometry of the trap and the TD does not change between 2017 and 2018, the time constant $\kappa$ is the same both years. This time constant is approximately twice the saturation time of the GV monitor, and is approximately the saturation time in both the RH and the DS monitors.

Assuming a constant incoming flux, $\Phi$, for some filling time, $T_F$, the trapped neutrons available to the monitor, $M$ can be described by an integral:

$$
M = \alpha \int_0^{T_F} \frac{\Phi}{\kappa} e^{(t' - T_F)/\kappa} \, dt'.
$$

(4.7)

In equation (4.7), an arbitrary proportionality constant, $\alpha$, relates the flux $\Phi$ and the time constant $\kappa$ to the trapped neutrons. Integrating equation (4.7) over the entire filling time reproduces the number of trapped neutrons given by equation (4.5). In actual data from UCN$\tau$, $\Phi$ is not necessarily constant and the fill is not continuous. However, in the case that the fill lasts much longer than the individual time constants of the pulses, $\Phi \sim \frac{dN}{dt}$. In this way, the integral can be replaced by a finite sum across all counts in a given normalization detector, now incorporating an exponential weighting:

$$
M = \alpha \sum_{t=t'} \frac{R(t')\delta(t')}{\kappa} e^{(t' - T_F)/\kappa}.
$$

(4.8)

In equation (4.8), the instantaneous rate at a given time, $R(t')$, has become an approxi-
imation for the number of actual counts. For actual neutrons, one can take \( R(t') \delta(t') \rightarrow 1 \) count, and reconstruct an exponential weighting this way. The limit \( \kappa \rightarrow \infty \), where the monitor never reaches saturation, reconstructs the non-exponential weighting case of equal weights for each monitor count. On a 1 s bin histogram width, a constant background rate, from section 4.2.3, and a fixed 16 ns monitor deadtime correction, which will be described in equation (4.15), has been applied. The MCS deadtime is significantly shorter than the actual pulse shape after passing through the discriminator, and so the deadtime correction does not provide a significant effect. Since the quantity of interest is a reconstructed monitor count, and not a direct count of UCN, for a finite \( \kappa \), \( \alpha = 1 \). An uncertainty on each reconstructed monitor count can be found by taking the square root of the sum of squares for each neutron on the monitor:

\[
\delta(M) = \sqrt{\sum_{t=t'}^{\kappa} \kappa^2 e^{2(t'-T_F)/\kappa}}. \tag{4.9}
\]

The model for fitting weighted counts does potentially shift the lifetime, and improve the overall fit. This difference can be found calculating a lifetime with “raw” integrated counts, involving no background subtraction or weighting, and comparing that to the appropriately exponentially weighted backgrounds. For a coincidence paired analysis, this can shift the lifetime by \( 0.45 \pm 0.03 \) s. This is a cumulative effect of both monitor weighting and background subtraction for monitor counts on estimating the initial counts. Since the SP and RHAC have non-negligible backgrounds, reconstruction of monitor counts must incorporate some background estimate. Various bad runs, described in section 4.3, can potentially skew the ability to reconstruct normalization counts. A component of the nonzero shift present here could potentially be due to poor runs remaining in the analysis. The principal gain from an exponential weighting comes from the gain in describing normalization.
4.3 RUN SELECTION

4.3.1 Overview

The 2017 run cycle began on July 25\textsuperscript{th} 2017, with data-taking beginning with run 3412. The end of the 2018 run cycle occurred on December 20\textsuperscript{th} 2018, with run number 14731. In between these two dates, the MCS data recording software logged 11319 total runs. Not all of these runs actually could be used for production; many of these runs featured systematic studies, backgrounds, or even simple debugging of various UCN\textsuperscript{τ} elements. A lifetime production run utilizes a specific UCN\textsuperscript{τ} movement pattern, previously described in section 4.1.2. Of these 11319 total runs, only 5590 were actually intended for production running and measurement of the lifetime.

Issues with beam stability during filling or actuator failures during transitions between trap states can lead to an unusual number of neutrons in the unload. These problematic runs should be excluded from the lifetime analysis due to their unpredictable nature, as these variations cannot be easily modeled. Thus, the first step towards finding the neutron lifetime, $\tau_n$, should be to remove any production runs that, due to experimental glitches, could add uncertainty and bias to the lifetime. Some of these unstable runs could potentially be used for background or normalization studies. With the addition of the RH monitor for 2018, the beam stability becomes less of an issue.

As a general principal, runs without counts in a detector or well outside the expected range of that particular holding time can be safely removed from analysis. Reconstructed neutron counts remaining in the holds can be seen in figure 4.6. With minimal run selection cuts, many runs with lower-than-expected counts can be seen. These runs typically come from either failure of movement, such as TD or GV failures, or depolarization from the holding field tripping off. Since they have no counts above background, they indicate hardware malfunctions, and signal regions where the UCN\textsuperscript{τ} trap has significant excess loss mechanisms. These runs clearly behave differently than normal production data.
Figure 4.6: Unload counts in the dagger detector by run. “Counts” have passed through a coincidence reconstruction algorithm, described further in section 4.4, to estimate the number of UCN remaining in the trap after the holding time. This algorithm requires 8 photons with a 50 ns initial window, a 1000 ns telescoping window, and a 1000 ns prompt window. The unloads follow the source outputs previously seen in the monitor counts, with the “sawtooth” behavior seen previously, scaled by an exponential decay based on holding time. In this plot, no quality cuts have been made to the data, so a class of runs with abnormally low counts for that holding time can be seen.

Various methods can be used to differentiate “good” and “bad” runs. The following section proceeds in the same order of operations by which “bad” runs are removed. First, section 4.3.2 describes how timing information is used to reject runs. Next, simplistic monitor count checks, described in section 4.3.3, are used to further remove bad runs. Issues with the two PMTs are demonstrated in section 4.3.4. Finally, a more rigorous outlier removal algorithm, described in section 4.3.5, removes the worst of the remaining runs.
4.3.2 Movement Tagbit Reconstruction

The UCNτ experiment features in-situ detection, as well as movement inside the trap with the trap door and cleaners. A failure by the trap door or cleaner to move would lead to UCN able to escape the trap during the hold. While many internal components cannot be directly seen, actuator movement can be recorded and reconstructed to determine timing of in-situ movements in the trap. These timings are recorded in the MCS in the form of “tagbits.” A tagbit in the data stream is an input channel that reads either “true” or “false.” Each event recorded by the MCS features an encoding for all of that box’s respective tagbits. This does limit the exact timing of the tagbit to the cumulative rate of background events in all channels, and thus leads to a bit of jitter at ∼ 1 kHz. A simple debouncing algorithm, requiring consistent tagbits for 0.2 s, compensates for this minor inconsistency.

Algorithm 2 Tagbit Debouncing

1: Starting time \( t_0 \)
2: for Events \( t_i, t_{i+1} > t_0 \) do
3:   if \( b_i(t_i) \neq b_{i+1}(t_{i+1}) \) then
4:     for \( t_j = t_{i+1}; t_j < t_{i+1} + 0.2 \) s do
5:       if \( b_i(t_j) = b_{i+1}(t_i) \) then
6:         Just jitter, break and continue from \( t_j \)
7:       else
8:         If the same for all \( b_i(t_j) \), record tagbit
9:       end if
10:  end for
11: end if
12: end for

Specifics of what these tagbits actually register depend on the particular channel of interest. Some tagbits characterize whether a GV is “open” or “closed,” or whether the Active Cleaner (AC) is “down” or “up.” Other channels instead correspond to motion, reading “true” while the dagger or TD is in motion. This allows the determination of dagger and cleaner movement timings during the run as well.

These tagbits are used to find the times at which the GV closes and the TD moves
at the end of the fill. Tagbits are then used to determine the holding time, $t$, as well as demarcations of the dagger dips and the background period at the end of the run. Additionally, since tagbits record actuator movement and stoppage, the tagbits can be used to isolate times where elements of UCN$\tau$ move, as a way to selectively remove regions with potential actuator noise. Runs where the tagbits are unreadable or where movement events occur out of expected order are excluded from production data. This could be indicative of cancelled runs or runs with hardware glitches; some of these are even incapable of being parsed by the analysis software.

Tagbit reconstruction does reject some potentially good runs. If channels in the MCS have changed, tagbits could be reading different and unexpected channels, which would appear out of order and fail this criterion. Due to the difficulty of manually hardcoding in holding and dagger movement timings for each run with bad tagbits, no attempt at reconstructing production runs without tagbits has occurred. Additionally, tagbits used for blinding the data failed during certain beam on and beam off configurations, leading to a small subset of production data excluded due to a blinding bug.

4.3.3 Fill Curve Fitting

A run that contains no data, or just background data, will have to be removed from the analysis. Spurious electrical noise can cause MCS channels to produce background counts, even with no UCN able to reach that detector. After checking fill timings, the next step is to remove runs with disconnected detectors or which contain runs where certain elements of the trap have not moved. UCN rates in a detector during filling, $R(t')$, can be simply modeled through an exponential fill up to some saturation constant:

$$R(t') = \Phi \left( 1 - e^{-t'/\eta} \right). \quad (4.10)$$

A “good run” and a “bad run” can be seen superimposed on one another in figure 4.7.
Figure 4.7: GV monitor rates during the filling time for a “good run,” run 4230, and a “bad run,” run 5052. In run 5052, the beam drops out at 20 s, causing the rate to drop off as the source stops producing UCN. Note the individual pulses perturbing the exponential fit, due to the pulse structure of the beam on target.

The ROOT software package’s histogram fitting algorithm determines the saturation constant, $\Phi$, and the filling time, $\eta$, for each run. A good run has a better fit than a bad run, with a $\chi^2$/NDF $\approx 3$. This function does not completely capture the source behavior, as the beginning of the fill has non-exponential components. Additionally, the pulse structure of the beam adds in a periodic fluctuation that is unaccounted for.

Calculation of either the $\chi^2$ or the time constants themselves for various monitors provides one method of finding “bad runs.” These methods, however proved to be too stringent, removing otherwise good runs from the analysis. A more simplistic test of fill quality checks if each run has exponential behavior in every detector. This can be done by forcing the $\Phi$ to be positive and $\eta$ to be between 5 s, the time between pulses,
and 5 s before the end of the trap’s filling time. The trap should be open for either 150 s or 300 s, depending on whether the RH is installed. The filling time has been chosen to reach UCN saturation in the trap, so if \( \eta \) takes longer than this, we have undesired behavior. This could perhaps stem from a beam recovery after a source arcdown or loss of beam during the fill.

As previously mentioned in section 4.2.3, some runs have a relatively high rate during the measured background period. Any runs with a monitor rate during the background period greater than 1% of the monitor rate during the filling time are removed at this time.

4.3.4 Light Leak Background Removal

Another potential source of bad runs that would not be described in the previous section would appear if a PMT glitches due to electrical noise or a light leak. These might have a normal filling pattern, and might not necessarily appear as a significant outlier, particularly under the influence of a coincidence algorithm, which will be described in section 4.4. Such an algorithm might reduce the effect of problems with a single PMT while glossing over an increased rate due to unanticipated backgrounds. For a single photon analysis, these irregular backgrounds would make a lifetime calculation impossible. In particular, such an event would lead to a non-negligible yield difference between two different methods of yield reconstruction, using coincidences or single photon counting.

A run with a light leak might not necessarily be unusable; a coincidence algorithm could be insensitive to potential shifts in the background. Nevertheless, a potential PMT glitch subtly affects the ability to reconstruct a coincidence, and can generate difficult to quantify second order shifts. To prevent any odd coincidence behavior, runs with a ratio between singles and coincidences less than 45 counts per coincidence or greater than 60 counts per coincidence are removed. The upper limit was chosen to sit above the main band of runs in 2017, seen in figure 4.8. Inspection of short runs with a counts per
Figure 4.8: Ratio of background subtracted singles yields to background subtracted coincidence yields by run. The expected yield sits between 45 and 60 photons per coincidence. Holding times longer than 2000 s, marked here with a black +, do not have high enough statistics to be reliable in a single photon counting method. Data in 2018 has more fluctuation due to fewer coincidence counts. Discrepancies between the two event reconstruction methods are indicative of light leaks.

coincidence greater than 60 showed clear time variations in the light yields. The lower limit of 45 lies below the main band of runs in 2018. The combination of this lower limit of 45 and upper limit of 60 provides a window of ±7.5 counts per coincidence to be classified as a “good run.”

Additionally, some nights of running with consistently elevated count rates are also removed if a significant number of runs in that region are near the light leak threshold. Light leaks cause the shift in relative yields from runs 11737 and 11982, as well as from runs 12649 to 12671 in 2018. Individual runs, in particular those with significant shifts due to background calculation, can appear as a bad run due to both a source issue and
due to a light leak.

4.3.5 Monitor Outlier Run Removal

In UCN\(\tau\), monitor detector outlier runs must be removed from the analysis so that there is no systematic bias when reconstructing the initial number of UCN in the trap. A run that has a surprisingly low or high number of monitor counts, \(M\), compared to the measured unload counts, \(U\), will get absorbed into whatever normalization algorithm is used, which could cause a potentially significant lifetime shift. Normalization methods will be discussed in section 4.6, but a simplified version can be used to remove outliers. Take a linear relationship between the unload counts and counts in a given monitor, \(U_k = \alpha_j M_{j,k}\). Given a constant holding time, \(t\), within constant running conditions, a quality factor for each monitor on each run, \(\Phi_{j,k}\), can be defined as:

\[
\Phi_{j,k} = \left| \frac{U_k}{M_{j,k}} - \langle \frac{U}{M} (t = t_k) \rangle \right| \langle \frac{U}{M} (t = t_k) \rangle .
\] (4.11)

Since large outliers can heavily shift the ratio of counts and thus bias the mean, equation (4.11) uses \(\langle \frac{U}{M} (t = t_k) \rangle\) as the median of the distribution. Since there are multiple normalization monitors, the combined quality factor for each run was then computed by taking the mean of the four main normalization monitors:

\[
\Phi_k = \left( \frac{\Phi_{GV,k} + \Phi_{SP,k} + \Phi_{(Bare/RHAC),k} + \Phi_{(Foil/RH),k}}{4} \right).
\] (4.12)

Taking the mean of these uncertainties provides a method of correspondence between various monitors, as various detectors could be more or less prone to beam-related glitches. A histogram of these run quality factors, divided into 2017 and 2018, can be seen in figure 4.9. “Bad runs” are defined as runs which appear outside the central distribution of run quality, in the secondary high peak. Since the quality factor for various detectors are highly coupled, the mean of the four quality factors could have nontriv-
Figure 4.9: Histogram of run quality factors, $\Phi_k$. The upper plot shows 2017 data while the lower plot shows 2018 data. In 2017, on-site shifters stopped “bad runs,” so many of these would fail the tagbit check. The presence of the RH in 2018 means that the peak of the run quality distribution is smaller, indicating better quality runs. However, due to changes in running procedures, a higher number of bad runs exist in 2018. Vertical bars indicate the cuts made, with runs to the right of the bars excluded from analysis.
ial statistical couplings. Calculation of this quality factor assumes that the underlying normalization distribution behaves identically between short and long runs, particularly with the same spread in monitor rates compared to dagger rates. For a true normalization, these assumptions could potentially lead to bias, as care has not been taken to provide identical weighting between short and long holding times. Any statistical bias evident in this quality factor, however, would only minimally affect the lifetime, since equation (4.11) does not get used for anything except run selection.

In order to maximize the statistical sensitivity, quality factor cuts were chosen to remove the worst runs while keeping a maximal amount of data. In figure 4.10, the lifetime has been calculated for each quality percentile, down to a minimum of 70 % of the available production runs. As the amount of data kept is reduced, the lifetime can potentially shift significantly due to poor modeling of the predicted number of neutron counts. However, including too much data could also potentially bias the lifetime by including poorly constrained runs. Because figure 4.10 utilizes a paired lifetime calculation, some potential systematic effects cancel, thus maintaining stability across a wide data set. A percentile cut of 97 % in 2017 and 93 % in 2018 was chosen. In 2017, this cut is immediately before the lifetime shifts outside of 1 σ. In 2018, the lifetime starts to change relatively rapidly above 96 %, so this cut sits in the middle of the region of stability immediately before this. These shifts are almost all within the increase in statistical uncertainty associated with adding additional data, and thus an associated systematic uncertainty would already be encompassed by statistics.

4.3.6 Run Breaks Generation

Not all production runs occurred with the same configuration of monitors. In between 2017 and 2018, the dagger was completely changed. This involved swapping out the PMTs, adding a thicker $^{10}$B layer on top of the Zinc Sulfide (ZnS:Ag) scintillator, and replacing some damaged fibers at the edge of the detector. Additionally, installation
Figure 4.10: Paired lifetime calculated for 2017 (top) and 2018 (bottom), using data above the given run quality percentile. The colored bands represent the 1σ and 2σ added uncertainty from the 70th percentile of run quality, which corresponds to the potential statistical fluctuation of added data. Adding more data to the lifetime calculation does not significantly statistically bias the lifetime, as the prior cuts provide the most important “bad run” parameters.
of the RH increased the filling time and drastically changed the incident UCN population passing through the guides. This necessitates a separate lifetime analysis of the two years. In order to isolate the effects of various monitor detector changes, data has been divided into 25 discrete slices. Each “Run Break (RB)” has different normalization parameters and potentially different background expected values.

Figure 4.11: Relative monitor counts on a run by run basis. Each monitor has been divided by the GV monitor counts. Discrete steps here correspond to either physical changes in the monitor locations or changes in the gain of various detectors. In 2017 the SP monitor maintains stability, while in 2018 the RHAC monitor moves up and down after installation.

Aside from the major change between the two years, minor changes in detector gains, cooling of the PMTs, and even major source reconditioning, change the relative counts in UCNτ’s detectors. Figure 4.11 shows the variation of monitor counts as a function of time, in order to show these shifts. These counts have all been scaled to the GV monitor, and as such discrete steps here signify a detector gain shift, or a physical change made.
in the trap, such as the RHAC raising or lowering. Dagger detector gain shifts can be found by comparing the single PMT background rates, and looking at times at which these shift. A full list of normalization RBs can be found in appendix A.

4.4 RECONSTRUCTION OF NEUTRON COUNTS

4.4.1 Overview

Neutrons, as a neutrally charged particle, present some difficulties for measurement. Most particle physics detection methods convert various particles into photons; neutrons are no different. Since UCN\(\tau\) does not directly measure the individual neutrons on the detector, reconstruction of probable neutron events must occur. In addition, the rates expected in the detector differ between short and long holds, as neutrons decay with time. Thus, any reconstruction algorithm produced must take into account any rate dependent effects that could lead to a change in efficiency.

In order to read out data and count neutrons on the scintillator, it is important to first understand various tools available to UCN\(\tau\). The MCS feeds four channels of dagger detector data into the data stream. Two of these correspond with PMT1, and two of these with PMT2. Each PMT registers photon pulses from both a low discriminator voltage threshold and a high discriminator voltage threshold. These set voltage thresholds vary slightly based on conditions in the area that might affect the efficiency of the PMT, with a goal of attempting to minimize PMT noise and maximize UCN generated photons. Event reconstruction then involves determining the efficiency of converting UCN to PMT counts, and investigating potential inconsistencies between short and long reconstructed neutron counts.

4.4.2 Single Photon Events

When a UCN hits the dagger, it can reflect off, capture on the \(^{10}\text{B}\) surface, or be lost. Rates of reflection and loss can be described quantum mechanically through the surface
properties of the capture layers, with relevant UCN properties being the incident angle and the UCN kinetic energy[13]. The physical thickness of the dagger layers, and thus the absorption and loss probabilities of UCN with a known angle and energy, do not change significantly on a run-by-run basis. Assuming the bulk phase-space distribution of UCN in the trap does not change significantly, the efficiency of neutron capture compared to neutron reflection and loss on the dagger does not change. A potential systematic uncertainty can be introduced due to changes in phase-space distribution between short and long holding times. This will be described later in section 6.6, and its accompanying appendix D. Even in the presence of potential phase space shifts, the $^{10}$B capture process provides the principal tool for determining UCN capture and UCN distributions in the trap.

Neutrons that capture on the $^{10}$B surface instigate the reaction $^{11}$B $\rightarrow$ $^7$Li + $^4$He, with the resultant ions having some additional energy. These $^7$Li and $^4$He ions produce scintillation light as they pass through the ZnS:Ag. This reaction produces the two ions back-to-back due to conservation of momentum, thus one of these two ions will always pass towards the detector. Each of these ions imparts some amount of energy into the detector in the form of photons, which then travel through the wavelength shifting fibers into the PMTs. After this process, each PMT records $\sim$ 20 counts from each UCN. The number of photons actually counted for each UCN capture event has to do with PMT efficiencies, detector geometry, and other potential unknowns. Over a large number of independent UCN events, the number of photons seen in the PMTs will converge to some mean with a non-Gaussian spread.

One possible avenue for measuring the neutron lifetime, $\tau_n$ would be to simply count the photons from each production run, as the number of photons counted is directly proportional to the number of UCN on the dagger. The total number of photons counted in each detector during production running can be seen in figure 4.12. In analogy to some particle detectors, a single photon counting method can be thought of as a “current
Figure 4.12: Single photon unload counts on a run-by-run basis for the sum of the two PMTs at low threshold. Different holding times show the expected drop in yields due to $\beta$-decay. The local “sawtooth” behavior comes from source conditioning, behaving similarly to the raw monitor counts. Discrete steps, such as at run 9600 and run 13209, occur when the PMT gains are adjusted or changes are made to the dagger detector.

mode” running of UCN$\tau$[67]. Since the MCS resolves events with 0.8 ns, a single photon counting mode would not suffer from event reconstruction inefficiencies, and the very short dead time would minimize rate dependent effects.

A single photon counting method, however, suffers from considerable backgrounds. Background reconstruction will be discussed in detail in section 4.5. Subtle changes in backgrounds, in particular potential position and time dependence in the background counts, limit the single photon counting method for constructing a lifetime.
4.4.3 Multiple Photon Events

A UCN captured by the detector produces many photons, which can be fed into either PMT. Importantly, the wavelength-shifting fibers of the dagger alternate to which PMT they connect. As a result, scintillation events should deposit energies into both PMTs, allowing a coincidence algorithm for neutron event identification. By requiring a photon to appear in each PMT, electrical noise from an individual PMT gets filtered out, as the dark noise rates in the two PMTs should be uncorrelated.

Each neutron produces a photon shower with more than 2 photons, so introducing a photon threshold \( \gamma_T \) within a window \( W_p \) can suppress PMT dark noise. If the dark rate \( R_D \) follows a Poisson distribution, and each PMT has an independent dark rate, \( R_{D1} \) and \( R_{D2} \), the probability of counting a background “coincidence” with \( \gamma_T \) counts in some window \( W_p \) is[68]:

\[
P(W_p; R_{D1}, R_{D2}) = \left[ 1 - \sum_{x=0}^{\gamma_T-1} \frac{[(R_{D1} + R_{D2})W_p]^x e^{-[(R_{D1} + R_{D2})W_p]}}{x!} \right].
\] (4.13)

Equation (4.13) is the sum of probabilities of any combination of Poisson counts over the threshold. Increasing \( \gamma_T \) reduces the probability of false events. However, even the act of requiring a photon hit on each PMT in a short initial window, \( W_I \), can suppress the background. The probability of two or more dark rate Poisson events triggering an initial potential coincidence within a given window is just the product of two non-zero events with mean rates \( (R_D W_I) \):

\[
P(W_I; R_{D1}, R_{D2}) = \left[ 1 - e^{-(R_{D1} W_I)} \right] \left[ 1 - e^{-(R_{D2} W_I)} \right].
\] (4.14)

These two probabilities can be used in combination to define coincidence events, requiring one photon on each PMT in a very short initial window and then increasing the threshold afterwards. To find an estimate of such dark noise based coincidence
events, look at the dark noise in each PMT. The backgrounds for 2017 and 2018 are roughly 150 Hz for PMT1 and 100 Hz for PMT2, depending on the specific background region. Choosing $W_I = 50$ ns, and forcing equation (4.14) to have at least one photon trigger, the probability of just starting a “coincidence” comes out to $\sim 7.5 \times 10^{-6}$. Then, following equation (4.13), a choice of $W_P = 1$ $\mu$s and $\gamma_T = 4$ gives a probability of $\sim 1.2 \times 10^{-16}$ for this triggered event to become an actual coincidence. While these parameters and background rates are free parameters, the overall effect of dark rate suppression remains generally valid. Particle events and non-independent PMT noise, such as electrical interference from movement of UCN trap elements, can still provide some potential sources for background in a coincidence mode.

4.4.4 Coincidence Overview

In order to increase the signal-to-noise ratio in our neutron counting, the principal lifetime analysis uses a coincidence algorithm, algorithm 3, to identify probable UCN events. After finding two photons in a short window, this coincidence algorithm looks at the interarrival time of subsequent neutrons and “telescopes” out to find trailing photons from the same event. The telescoping window attempts to absorb as much of the long tail of photons generated by a UCN as possible.

The method described in algorithm 3 features a handful of parameters, which can be modified to modify rate dependent effects. Typical coincidence timing parameters are the initial window $W_I = 50$ ns, the prompt window $W_P = 1000$ ns, and the telescoping window $W_T = 1000$ ns. A normal photon threshold is $\gamma_T = 8$ photons. After finding a coincidence, individual coincidence data is reduced to its length, position, and number of photons counted on each PMT.

---

2The “photon threshold” $\gamma_T$ as used here is defined as the number of “photons” found in the event. An additional “threshold” is used in the analysis, as the low and high PMT discriminator thresholds vary the the voltages required to generate a “count.” The photon threshold is the relevant definition when discussing coincidences and event reconstruction, as the discriminator thresholds are fixed during each run.
Algorithm 3 Coincidence Finding.

1: Number of photons $\gamma = 0$
2: Photon registers on a PMT at time $t_0$, record which PMT it hit
3: Look for the next photon that hits a PMT at time $t_i > t_0$
4: if $(t_i - t_0) < W_l$ then
5:   if $t_i$ hit the same PMT as $t_0$ then
6:      $\gamma = \gamma + 1$; find next photon and increment $t_i$
7:   else if $t_i$ is the opposite PMT to $t_0$ then
8:      $\gamma = \gamma + 1$, find next photon and increment both $t_i$, $t_j$
9:   while $(t_j - t_i) < W_T$ do
10:      if $(t_j - t_0) < W_p$ then
11:         $\gamma = \gamma + 1$
12:   end if
13: end while
14: if $\gamma \geq \gamma_T$ then
15:   This is a coincidence. Repeat algorithm with next photon after end of loop.
16: else
17:   Not a coincidence. Repeat algorithm with next photon after $t_0$
18: end if
19: end if
20: end if

The coincidence algorithm still includes some background events, due to particle interactions with the detector. As can be seen in figure 4.13, the number of photons counted in a UCN event can be seen in a broad region, with an eventual long tail presumably due to backgrounds above $\sim 100$ counts. Coincidence background structures appear in two regions, one falling off relatively quickly and one having a large number of counts inside the region. Unfortunately, aside from specific obvious cases, background particle events can be difficult to distinguish from neutron counts. One of these special sources of backgrounds will be discussed in section 4.4.8.

The scintillation initiated by neutron events has a long time structure, on the order of $\sim 10 \mu s$. These time structures can be seen in figure 4.14, and take a significant amount of time, $> 10 \mu s$, to reach backgrounds. Early photon instabilities, such as a 120 ns afterpulse or the 16 ns deadtime, cause non-exponential behavior early in the event. The long length of the coincidence necessitates the telescoping window; single UCN
Figure 4.13: Summed foreground and background coincidence pulse heights, with data taken from the end of 2017. Background rates, taken from the dedicated “background” period at the end of production runs, have been scaled by the signal-to-background ratio. Foreground events have a broad peak, with $\sim 30$ photons counted in each coincidence. Background events fall off rapidly, before becoming a broad, relatively flat distribution.

Events should try and absorb as much of the tail as possible. If too few photons from a coincidence get counted, the tail could potentially lead to multiple triggers on a single UCN. These coincidence timing effects depend on the rate of counts in the detector, and can significantly change the lifetime measured by UCN$\tau$.

4.4.5 Limitations of High Threshold Counting

Raw data from the PMTs is shaped and triggered with two different discriminator thresholds, low and high. These thresholds are tunable voltages above which the PMT registers a “count.” These two channels should, up to some amount of noise, generate the same yields and backgrounds. The higher discriminator setting would ideally remove some of
Figure 4.14: Photon rates as a function of time for photons in “coincidence” events in PMT1 and PMT2, for the end of 2017. Foreground structure samples from low rate sections of the unload, only accepting UCN with a separation of 50 $\mu$s or greater. Background structure samples from the end of the run with the same timing constraints, with relative probabilities scaled by the expected signal-to-background of the entire unload. Structure at the beginning of the event comes from PMT deadtime and afterpulsing, while the long glow comes from the ZnS:Ag scintillation time.

the lower energy background noise, thus providing an independent check of the background model. Most of the MCS counts will be identical, ideally with some constant shift in efficiency between high and low thresholds. The “principal” data channel for this analysis, however, must be the low threshold events, due to inconsistencies in the high threshold.

As can be seen in figure 4.15, discrete changes in the high threshold counting shift by $\sim 1$ PE various times during the 2017 running. These discrete changes appear to be a consistent single photon gain drift, possibly due to discriminator temperature response.

These changes are not, however, due to high rate saturation of the PMT. As can be
Figure 4.15: Ratio of single photon yields to coincidence yields for 2017, calculated for both low discriminator threshold (upper plot) and high discriminator threshold (lower plot). The high threshold coincidence method contains discrete, single photon shifts in the PMT response, while the low threshold has a stable response.

seen in figure 4.16, the average number of photons counted in PMT1 in a coincidence does not depend on the local rate. If the PMT had a high rate saturation, the response would vary by more than a photon between high rates, after a $S$ holding time, compared to the lower rates after $L$ holding times. As the two thresholds have a similar response to varying rates, the gain drift is more likely a hardware issue. As a result, the best procedure is to separate the PMT gain shifts as additional run break sections, and focus on the low threshold counting.

4.4.6 Deadtime

The measurement efficiency of neutrons in UCN$\tau$ couples directly to the rate at which neutrons hit the dagger. Two competing effects shift the number of “neutrons” at a
Figure 4.16: Average number of photons counted in PMT1 in each coincidence event during the unload. Data was taken during 2017 and is indicative of PMT instabilities. The top plot displays the number of photons counted for both a short and long holding time, while the bottom plot shows the difference between rates for both low and high PMT thresholds. There is no evidence for the average amplitude of coincidence events changing as a result of detector rates.

given rate. “Deadtime” of the detection algorithm reduces the detection efficiency, while “pileup” causes under-threshold events to trigger in our model. Accurately measuring the neutron lifetime requires correcting the detected number of events to the actual number of events.\(^3\)

A radiation detector such as our dagger PMTs cannot discriminate 2 or more events occurring in a short time. Real systems typically have a combination paralyzable dead-time, where subsequent events can push back the deadtime, and non-paralyzable dead-

\(^3\)For more information on this topic, Knoll’s book “Radiation Detection and Measurement” has a thorough overview on various rate-dependent effects in particle detectors[67]. Much of this section will follow from discussions in there.
time, where all events have the same amount of deadtime. The time after an event
during which our detector does not resolve subsequent events is the deadtime, which in
the non-paralyzable limit can be corrected as:

\[ R_{\text{corr.}} = \frac{R}{1 - R\kappa_0}. \] (4.15)

In equation (4.15), \( R_{\text{corr.}} \) is the actual rate, \( R \) is the measured rate, and \( \kappa_0 \) is the
deadtime. In UCN\( \tau \), the discriminator deadtime in our dagger PMT, used for singles
analysis, is non-paralyzable with \( \kappa_0 = 16 \pm 0.8 \) ns. This deadtime is found by looking
at the minimum time to next event in a given PMT, with uncertainties of one MCS time
tick. In singles analysis, the number of photons counted is first summed into 0.1 s bins,
and then scaled using the measured rate, \( R \). This leads to a deadtime correction required
for a singles lifetime measurement of 0.16 \( \pm \) 0.01 s, combined across the 2017 and 2018
datasets.

For a coincidence analysis, however, the telescoping window creates a paralyzable
deadtime. The real rate, \( R_{\text{corr.}} \), can be determined from the measured rate, \( R \), using:

\[ R = R_{\text{corr.}} e^{-R_{\text{corr.}}\kappa_0}. \] (4.16)

Equation (4.16) cannot be solved analytically, so properly accounting for deadtime re-
quires an iterative or Monte Carlo based approach. As this would be too computationally
extensive to solve for every single measured rate for every single run, in practice the par-
alyzable deadtime can be approximated as non-paralyzable. In the limit \( R_{\text{corr.}} \ll \frac{1}{\kappa_0} \), both
paralyzable and non-paralyzable deadtime corrections can be expanded to a quadratic:

\[ R \approx R_{\text{corr.}} (1 - R_{\text{corr.}} \kappa_0). \] (4.17)

Because our maximum measured coincidence rate is on the order 10^4 Hz, while the
average length of a coincidence is of order 10^{-5} s, using a non-paralyzable deadtime
across many different runs should be close to the appropriate deadtime. Binning the data to 0.1 s sections, the deadtime can be estimated by calculating the average length of coincidence, $\kappa_\mu$, and using this as the effective deadtime.

### 4.4.7 Pileup

The ZnS:Ag used as a neutron detector in UCN$\tau$ has a long characteristic lifetime, on the order of $\sim 10 \, \mu s$. This characteristic glow is a problem; the intrinsic PMT dark rate has a higher probability of forming false coincidence events shortly after a UCN event. Photons from the tail of previous events can boost the current event above the counting threshold, increasing the probability of detection. Additionally, the long deposition time of photons in the dagger means that photons from a single neutron event could potentially trigger multiple times. These can be exacerbated by making poor choices for the threshold and integration window.

A UCN hitting the detector will produce photons following a time-dependent probability distribution function $P(t)$. For a given event, the coincidence algorithm records $\gamma$ photons with total coincidence length $\kappa_0$. This algorithm will miss some photons from the total amplitude, $A$. On average, this amplitude will be the same for all UCN in a given detector condition, which has been previously demonstrated in figure 4.16. The total amplitude of a single event can be estimated through the integral equation:

$$A = \frac{\gamma}{\int_0^{\kappa_0} P(t) \, dt}. \quad (4.18)$$

The mean number of photons, $\mu$, expected to occur between two times $t_1$ and $t_2$ after the initial UCN event can be written as:

$$\mu = A \int_{t_1}^{t_2} P(t) \, dt. \quad (4.19)$$

Each UCN that hits the detector afterwards will have some additional glow due to
the previous event’s probability distribution. This will affect an arbitrary \( m \)th UCN, triggering at time \( t_m \), after \( n \) other UCNs, which each hit the detector at \( t_n \). The number of excess pileup photons from previous events, \( \mu_m \), can be determined by summing \( \mu \) from all previous events within that event’s coincidence length \( \kappa_{0,m} \). Assuming each neutron event has the same total amplitude \( A \), this excess will be given by:

\[
\mu_m = A \sum_{i=0}^{n} \int_{t_n - t_i}^{(t_m + \kappa_{0,m}) - t_i} P(t) \, dt.
\] (4.20)

Unfortunately, solving equation (4.20) independently for each event is computationally expensive. To speed up calculation, we recognize that the previous summed integrals each modify the total amplitude \( A \) for later events. This means that we can recursively approximate the number of excess pileup photons on the \( m \)th coincidence as:

\[
\mu_m = (A + \mu_{m-1}) \int_{t_m - t_{m-1}}^{(t_m + \kappa_0) - t_{m-1}} P(t) \, dt.
\] (4.21)

This is not a perfect assumption, as it requires certain forms of \( P(t) \). Maximum coincidence rates in the detector are \( \mathcal{O}(10^3) \) Hz, while each event has a long tail of length \( \mathcal{O}(10^{-5}) \) s. The interarrival time of UCN in the detector should be spaced enough that the long tail will be dominated by the immediate predecessor. In the event that the long-term pulse structure is monotonically decreasing, equation (4.21) should behave appropriately.

A correction due to the pileup effect can be found by de-weighting each coincidence \( m \) by the probability of \( j \) photons forming a false coincidence. Using Poisson statistics, the probability of \( \gamma \) events occurring in a window with mean number of events \( \mu \) is:

\[
P(\gamma) = \frac{\mu^\gamma e^{-\mu}}{\gamma!}.
\] (4.22)

In the event that the spacing between coincidences is significantly longer than the length of the ZnS:Ag glow, the excess photons due to previous events will approximately
behave as a Poisson distribution with mean $\mu_m$. The probability $f_m$ of a coincidence with threshold $\gamma_T$ being real can then be found by summing equation (4.22) over all counted photons $\gamma_m$. Each coincidence event can then be assigned a weight based on the expected excess pileup determined from equation (4.21):

$$f_m = 1 - \sum_{i=0}^{\gamma_m-\gamma_T} \frac{\mu_m^i e^{-\mu_m}}{i!}.$$  \hspace{1cm} (4.23)

In reality, equation (4.23) will slightly over-estimate the correction required since $\mu_m$ decreases over the length of the coincidence. Nevertheless, naively calculating over all possible combinations of excess photons and times would take $O(N!)$, which is computationally unfeasible.

Each coincidence’s length $\kappa_0$ and number of photons $\gamma$ has already been recorded, and so these can be used as input variables in the model. Utilizing equations (4.18) and (4.21), the weighting in equation (4.23) for each event can be evaluated given some determination of $A$ and $P(t)$. This can be done by making some simplifying assumptions about the structure of events, and calculating $A$ and $P(t)$ directly from the coincidence structure of recorded events. In the absence of afterpulsing, $P(t)$ is monotonically decreasing. Assume that $P(t)$ can be written as the sum of $j$ exponentials with time constants $\tau_j$ and scaling factors $\kappa_j$:

$$P(t) = \sum_j \kappa_j e^{-t/\tau_j}.$$  \hspace{1cm} (4.24)

This has the advantage of being analytically integrable, and appropriately goes to 0 as $t \to \infty$. Short-term irregularities such as multi-pulsing or deadtime will cause non-exponential behavior. For pileup effects, we are only interested in the long-term structure, as these fast effects will occur during the initial coincidence window. Since
this is a probability distribution, we can constrain our variables using:

\[
\int_0^{\infty} P(t) \, dt = \sum_j \kappa_j \tau_j = 1. \tag{4.25}
\]

The average amplitude, \(A\), can be determined by looking at the total excess in photons during the unload. There are two types of photons counted in the dagger: single PE noise and photons from potential coincidence events. Coincidence events here might not necessarily be UCN, as some particle backgrounds will also trigger the coincidence algorithm. For an unload with \(x\) total photon counts, \(y\) background counts, and \(z\) recorded coincidences, the average number of photons per coincidence is simply:

\[
A = \frac{x - y}{z}. \tag{4.26}
\]

Then, for an event with \(\gamma_m\) counted photons and a length \(\kappa_0\), using equations (4.24) and (4.26) in equation (4.18) gives:

\[
\frac{x - y}{z} = \frac{\gamma_m}{\sum_j \kappa_j \tau_j(1 - e^{-\kappa_0/\tau_j})}. \tag{4.27}
\]

The next step is to determine \(\kappa_j\) and \(\tau_j\) for however many exponential terms we want. An analytic solution to equation (4.27) can be found if \(P(t)\) features only a single “effective pileup time” constant \(\kappa_{PU}\). Actual UCN events have more than one exponential; this model is an oversimplification. However, the most significant deviations from exponential behavior occur at short time scales. This simplistic model works if the telescoping window \(W_T\) is long enough to absorb most of of the non-exponential behavior. In the event that \(W_T\) is too short, this model would undershoot the effect of pileup. The recorded \(\gamma\) and \(\kappa_0\) values for all coincidences in an unload can be averaged to find the average number of photons, \(\gamma_\mu\) and the average coincidence length \(\kappa_\mu\). Utilizing these
averaged values, equation (4.27) can be rearranged to solve for \( \kappa_{PU} \):

\[
\kappa_{PU} = \frac{-\kappa_{\mu}}{\ln \left(1 - \frac{\gamma_{\mu} x}{x - y}\right)}.
\]  

(4.28)

The effective pileup time constant \( \kappa_{PU} \) and the amplitude \( A \) can be calculated on a run-to-run basis. The results of these calculations can be seen in figure 4.17.

With a determination of \( A \) and \( \kappa_{PU} \), the next step is to loop through the coincidences and calculate the weighting for each. The excess photons for each coincidence event can be calculated using equation (4.21), and then the coincidences can be re-weighted through equation (4.23). The relationship between the number of raw counts in an unload and the correction due to pileup can be seen in figure 4.18. Each event has a reduced probability of being a “real” event; the total number of recorded counts will be reduced by some small fraction.

Equation (4.28) used a single exponential to find an analytic solution to equation (4.27). However, an arbitrary number of additional exponentials can be added to the probability function \( P(t) \). Such higher order expansions would require additional recorded values from each coincidence event. Each additional exponential added requires two additional constraints from that run’s average coincidence data. This could be done by calculating the average time at which \( PE = 2, 3, \ldots \) is counted. Alternately, fixed time windows of \( \kappa_0 = 500, 100, \ldots \) ns could be used, with the average number of photons at these times counted. Such additional constraints would no longer be analytic, and would thus require numerically solving for the probability function.

An alternate way of determining the pileup correction would be to read through the data, fill up pulse height structure histograms, and calculate these directly for each running condition. Then \( A \) and \( P(t) \) could be read in from generated histograms. The disadvantage of this is that we have to choose some coincidence parameters to create pulse height structure histograms. Understanding our rate dependent effects would
Figure 4.17: In-situ measurement of $A$, above, and $\kappa_{PU}$, below, on a run-by-run basis. Variations in the amplitude can be due to discriminator settings and single photon backgrounds. The $\kappa_{PU}$ remains consistent within $\sim 100$ ns across 2017 and the beginning of 2018, as the ZnS:Ag structure does not change. The change at run 13309 corresponds to the breaking of PMT1, which changes the underlying background model. Such a shift stems from a significant change in the efficiency of counting photons in a coincidence. This results in a substantial shift $\kappa_\mu$ and $\gamma_\mu$. 
Figure 4.18: Comparison of raw unload counts $U$ to counts “removed” due to the long tail correction with a single exponential. With higher rates, more events will be deweighted due to pileup. This effect affects roughly 0.1% of the total counted UCN. Long holding times have fewer counts and are thus less affected by pileup effects.

require understanding the effect of these input parameters on the output data, which might have nontrivial effects on the correction. These extensions to the pileup correction could provide valuable complementary information to the analytic model described here. Such modifications of $P(t)$, $A$, and $\mu_m$ might be able to more accurately model the pileup. However, these would require further study and potentially need additional computing resources.

The combined effect of deadtime and pileup on a coincidence analysis, using a 1000 ns telescoping window after a 50 ns initial window and a 8 PE threshold, is a lifetime shift of 1.16 s. As rate dependent effects can shift the measured lifetime by a significant amount of time, they add systematic uncertainty to the lifetime measurement. A verification of the model used here will be described in section 5.3. The systematic
uncertainty due to this model can be determined in section 6.4.

4.4.8 Coincidence Timing Structure

Particular classes of particle backgrounds appear with unique coincidence parameters. The two PMTs in UCN\(\tau\) only read a binary energy signal, either above or below a certain threshold. This unfortunately limits the ability to adequately determine many of the potential background events in coincidences.

Some coincidence backgrounds can be investigated by looking at the balance of photons appearing in the two PMTs. A UCN event, absorbing into the \(^{10}\text{B}\) detector layer, should create a signal in both PMTs. As can be seen in figure 4.19, a class of background events imparts nearly all of their energy into one PMT or the other. These particular events are very difficult to separate from a “real” UCN event, as the pulse height structure for “real” UCN has a very broad distribution.

A more easily separable background can be seen by combining the coincidence length, \(\kappa_0\), with the total number of photons counted, \(PE\). A category of “fast” backgrounds appear along the \(\kappa_0 = PE \times (16\ \text{ns})\) line. This corresponds to events triggering in a PMT at the fastest possible rate; recall that 16 ns is the deadtime of each PMT. These “fast” events could be Cherenkov events from high energy radioactive decays in the area, or maybe even electrical noise in the PMT, but nevertheless can be safely removed as non-neutron events. Unlike with the previously described single-PMT coincidence backgrounds, a significant gap appears between these background events and the expected foreground events. As a result, a “fast” event is defined as any coincidence event with:

\[
\kappa_0 \leq PE \times (24\ \text{ns}).
\]

An additional possible constraint that does not remove any UCN from the data can
Figure 4.19: Pulse height distribution of all coincidence events, selected from data taken during the end of 2017. This 2D histogram records the number of coincidence events with a given number of photons in PMT1 and PMT2. The upper plot contains the foreground, while the lower plot shows the background events. Some background events trigger primarily in one PMT, while actual coincidence events appear in both roughly evenly, with a very broad distribution.
Figure 4.20: 2D Histogram showing the length of the telescoping window compared to the total number of photons counted in that event, for the end of 2017. The upper plot shows events during the counting, while the lower plot only counts the background events. A number of “fast” coincidences with lots of photons appear along the PMT deadtime. A cut can be made by removing any coincidence event that saturates the detector, chosen as $1.5 \times$ the MCS deadtime of 16 ns. An additional potential cut, which does not remove UCN events, offsets the cut by the coincidence threshold.
be achieved by offsetting equation (4.29) by the photon threshold, $\gamma_T$:

$$\kappa_0 \leq (PE - \gamma_T) \times (24 \text{ ns}).$$  \hspace{1cm} (4.30)

This modification would only remove the “fast” events, and not cut any UCN events. However, the actual presence of “fast” events during the unload can be difficult to determine, especially during high rates.

Figure 4.21: “Fast background” rates summed over the unload after 20 s holding times. Vertical lines indicate dagger and trap door timings. Two possible models for a fast coincidence have been plotted, one with no offset for the minimum threshold of a fast event and one without. UCN events during counting can appear at the same time as fast events, suppressing their rate.

Rejecting “fast” events from any coincidence algorithm does not fully remove the background. “Fast” events only provide a small change in background, peaking at around 0.01 Hz. As seen in figure 4.21, many of these events occur during the time
where we would like to estimate background rates, and thus could bias the background model for that run. Additionally, some of these appear to be generated by UCN events, as the rate of fast events immediately following the unload appears slightly elevated. There does not seem to be a systematic difference between “fast” backgrounds in short and long holding times, and so either cut can safely be applied. Nevertheless, since these background events can be easily removed, it makes sense to do so. Convolution of pulse height asymmetry with additional timing information could prove an avenue for future research, potentially suppressing backgrounds even further, although this has not yet been investigated in depth.

4.4.9 Single PMT Integrated Window

The coincidence algorithm exists to reduce the effects of backgrounds on the analysis. As the dark rate events should behave independently for each PMT, the requirement for counts to appear in both detectors simultaneously suppresses dark rate events. However, as discussed in section 4.4.3, the addition of a higher photon threshold in equation (4.13) provides many orders of magnitude more dark noise suppression than the requirement of an initial window. As a result, an alternate method of analyzing the data would involve an integrated window using just a single PMT. This would still heavily suppress backgrounds, and provide independent checks of the two PMTs. A modification can be made to algorithm 3 to produce an “integrated window” analysis.

An integrated window analysis has the same issues with deadtime and pileup as a typical coincidence event. The integrated window reconstructed counts must then be corrected for rate-dependent effects through the same procedure as in section 4.4.6 and section 4.4.7. As a result, they do not make a very good systematic cross check of the coincidence lifetime.

However, such an analysis allows studies of the independent PMT response without worrying about the backgrounds, which can be a limiting factor in lifetime measure-
Figure 4.22: Integrated window pulse height and timing structures. Data taken from the end of 2017. The upper plot shows PMT1, while the lower plot shows PMT2. Note that the “fast” coincidence background appears in the integrated window events as well.

1: Number of photons $\gamma = 0$
2: Photon registers on PMT$\alpha$ at time $t_0$
3: Look for the next photon that hits PMT$\alpha$ at time $t_i > t_0$
4: if $(t_i - t_0) < W_I$ then
5: $\gamma = \gamma + 1$; find next photon and increment $t_j$
6: while $(t_j - t_i) < W_T$ do
7: if $(t_j - t_0) < W_P$ then
8: $\gamma = \gamma + 1$
9: end if
10: Increment both $t_i, t_j$
11: end while
12: if $\gamma \geq \gamma_T$ then
13: This is an “event”. Repeat algorithm with next photon after end of loop.
14: else
15: Not an “event”. Repeat algorithm with next photon after $t_0$
16: end if
17: end if

ments. As can be seen in figure 4.22, the pulse height and timing information does not indicate that “fast” events are limited to one bad PMT. Rather, the saturated events can appear in either PMT with a similar probability.

4.5 BACKGROUNDS

4.5.1 Overview

Neutron event reconstruction in UCN$\tau$ is complicated by the presence of various sources of backgrounds. PMTs have an inherent dark rate, and the ZnS:Ag scintillator of the dagger glows for a very long time after being activated. These backgrounds, usually generating single-photon events, couple to temperature and ambient light leaks inside the UCN$\tau$ apparatus. Additionally, energetic particles such as cosmic rays or radioactive decays in the experimental area can induce scintillation events in the dagger above the coincidence threshold. An accurate measurement of the neutron lifetime requires precise accounting of these background effects.

The reconstructed number of counts in our detector, $Y$, will be modified by the back-
ground for a given run. A short holding time has a higher signal, and assuming a constant background rate $B$ thus has a higher signal-to-noise ratio than a long holding time. Assuming backgrounds between short and long runs are consistent, a paired lifetime will be shifted by:

$$\tau_n + \Delta(\tau_n) = \frac{t_L - t_S}{\ln \frac{Y_S - B}{Y_L - B}}.$$  \hspace{1cm} (4.31)

Taking as exact the knowledge of timings and yields, and expanding about a small offset in the background, $\bar{B} \rightarrow 0$, the lifetime shift due to backgrounds becomes:

$$\tau_n + \Delta(\tau_n) = \frac{t_L - t_S}{\ln \frac{Y_S - B}{Y_L - B}} + \frac{(t_L - t_S) (Y_S - Y_L)}{(Y_S - B) (Y_L - B) \ln^2 \left( \frac{Y_S - B}{Y_L - B} \right)} \langle B - \bar{B} \rangle \left[ \ln \left( \frac{Y_S - B}{Y_L - B} \right) \left( \frac{1}{Y_L - B} - \frac{1}{Y_S - B} \right) \right. $$

$$+ \left. \left( \frac{1}{Y_L - B} + \frac{1}{Y_S - B} \right) \right] \langle B - \bar{B} \rangle^2 $$

$$= \tau_n + \tau_n \frac{(Y_S - Y_L)}{Y_S Y_L \ln \left( \frac{Y_S}{Y_L} \right)} \left[ \ln \left( \frac{Y_S}{Y_L} \right) \right. $$

$$\left. \left( \frac{2 (Y_S - Y_L)}{Y_S Y_L \ln \left( \frac{Y_S}{Y_L} \right)} - \frac{Y_S - Y_L}{Y_S Y_L} \right) \right] \delta(B)^2. $$ \hspace{1cm} (4.32)

Without an appropriate background correction, a background rate of 1% of the short holding time unload signal will shift the lifetime by about 24 s, obviously well outside the required precision of $UCN\tau$. The necessary precision of background measurements can be found through equation (4.32). For an estimated neutron lifetime, $\tau_n = 880$ s, and choosing $Y_S = 1$, a variation of of 0.1 s comes from a shift in the backgrounds of merely $4.21 \times 10^{-5}$ signal in the short holding times. In a coincidence counting mode where the unloads are $\sim 10000$ UCN during a 210 s counting time, the lifetime will be shifted outside our desired range of precision with just a 0.002 Hz variation in the backgrounds!
Single PMT Backgrounds at Bottom of Trap

Background rates in single PMT events can be seen in figure 4.23. In 2017, background rates sit at around \( \sim 100 \text{ Hz} \) for both PMTs, while in 2018 damage to PMT1 near the end of the dataset reduces the single photon rate. Single photon rates fluctuate significantly based on ambient qualities such as temperatures and potential light leaks. Spikes in the background rate occur whenever maintenance of trap elements necessitate opening the vacuum chamber, as it takes a non-negligible amount of time for the PMT to cool down to normal running temperature.

Coincidence background rates, as seen in figure 4.24, are significantly more stable than the single photon rates in figure 4.23. The lower background rates mean that statistical precision becomes an important contribution to background uncertainties, as a short 50 s background measurement time could potentially not provide enough information.
Figure 4.24: Coincidence background rates at the bottom of the trap on a run-by-run basis. Background rates during production runs only use the last 50 s of the hold, while background rates for dedicated background runs use the entire time available. Production backgrounds, with their limited counting time, are quantized by the limited number of events counted in the dedicated background counting time. This effect is not seen in dedicated background runs.

While single photon counting methods become limited due to background fluctuations, the methods described in this section apply to both single and coincidence background calculations. As single photon counting methods are significantly more susceptible to background effects, such a single photon analysis helps model potential deficiencies in backgrounds.

4.5.2 Calculation of Raw Backgrounds

A normal production run contains a handful of regions where no neutrons should be counted. At the end of each run, the trap door opens for the last 50 s in order to measure
the lifetime on a run-by-run level. As the trap door is open during this time, this section
does not contain any neutrons, which would escape the trap and be counted in the DS
monitor. However, the motion of the TD potentially induces additional noise, and could
potentially add light leaks to the data. Using the tagbits, the motion of the trap door
for this section can be gated off for background calculation. One TD and CD cycle takes
approximately 10 s, meaning 40 s are available for background calculation.

As the counting period keeps the dagger at the bottom of the trap for 150 s, the
end of the counting period could provide additional background data. Of course, this
potentially would include neutrons in the background period leading to a bias. Finally,
as neutrons should have been cleaned and thus incapable of interacting with the dagger
during the hold, the holding time can be investigated as a potential avenue for further
background measurement.

A difficulty from using any of these times is the amount of time measured. As the
background rates for a coincidence event is below 1 Hz, the expected number of counts
measured in the last 50 s will be on the order of tens of counts, potentially subject to sta-
tistical biases. Singles counts, with background rates two orders of magnitude higher, do
not suffer as significantly from these biases. Comparisons between these measurement
periods, using singles counts to maximize statistics, can be seen in figure 4.25.

These background periods give insight into some of the physical mechanisms for var-
ious forms of backgrounds. The rates measured during the holding period in particular
are offset by a couple of percent. This is due to height dependent backgrounds, and will
be discussed more in section 4.5.3. This percentage changes throughout the data set, as
cooling and dagger configurations change. As a result, it is important to separate back-
ground regions where the detector configurations change. In 2017, a discrete step in the
background ratio occurs due to an unknown reason at run 7612. At this time, the height
dependence of the background in the dagger changes. As the dagger was completely
replaced between 2017 and 2018, the two years must be treated differently as well. In
Figure 4.25: Ratio of counts measured during the dedicated background period, at the end of run, with other regions without neutron counts. The left plot shows the ratio between the end of run and holding times, while the right shows the ratio between the end of run and end of counting. As the dagger sits in a different position during the holding time, the rates measured during the hold will be shifted by position-dependent effects. The rates seen during the end of the unload do not have this additional position dependence.
2018, at run 13307 the experimenters replaced PMT1. Although this does not cause not a significant shift with regards to position dependence, this section of data similarly must be treated differently than the rest of 2018.

Figure 4.26: Fitted draining time to the coincidence unload and the PMT2 unload in the end of 2017. The draining time is fit to the sum of 2 exponentials plus a background. Vertical lines at 430 s and 440 s gate off TD motion. The backgrounds in the last 50 s of the unload are consistent with the backgrounds at the dedicated background period. The single PMT reaches the ambient background slightly faster than the coincidence counting.

In the event that UCN still have not been counted, using the last 50 s of the unload would lead to a systematic bias in the lifetime. The draining time of the last unload dip can be seen in figure 4.26. The summed unloads during a constant background period can be fit to the sum of two exponentials and a constant background. Then, the background rate in the last 50 s of the unloads can be compared to the backgrounds at the end of the run to determine the amount of additional bias. Using a double-
exponential, the mean background rate is $\sim 10^{-4}$ Hz higher than the fitted, which would not bias the lifetime outside of our range of interest. In 2017 there are two time constants of about 3 and 8 seconds, with statistical variations between runs. In 2018 the short time constant is around 3 seconds, but now the longer time constant is approximately 6 seconds.

Individual backgrounds can be calculated on either a run-by-run basis, using the dedicated background period and the last 50 s of the unloads, or kept constant across an entire RB. The variation between these two cases provides the statistical limit of the knowledge of our backgrounds. In the case of coincidences, calculating a lifetime with either a constant background or varying by run shifts the lifetime by $0.08 \pm 0.01$ s. This comes mainly from an inability from a single run to adequately know the background counting model, and thus should not contribute to the uncertainty beyond what is already in the statistical uncertainty. For singles, as the single photon events have significantly higher fluctuation, the lifetime has a significant shift, of $0.76 \pm 0.03$ s. The extreme size of this uncertainty indicates a large lack of knowledge in the underlying single photon model for a given run.

4.5.3 Position Dependent Backgrounds

Backgrounds contain a position dependence, with dagger rates being measurably different depending on the position in the trap. Various potential sources of position dependence in UCN$\tau$ have been proposed. As the dagger physically moves throughout the trap, the efficiency of the PMT cooling tubing changes. This leads to potential dark rate gain shifts as result of temperature efficiency shifts. Additionally, the PMTs in the trap, and the scintillating surface, couple to potential ambient light leaks inside the trap. As the dagger raises and lowers, geometric factors could expose the scintillators to different levels of light or reflections. Finally, neutrons in the trap and other sources of radiation in the area can activate surfaces near the trap. As the dagger moves closer to the trap
surface or further away from the vacuum shielding, activated material can provide extra sources of background.

Figure 4.27: Summed dedicated beam-off daytime background, from the end of 2017 after the cooling changes. For each 250 s section, the dagger sits at a different height. The first 250 s, the dagger sits at 38 cm, then moves to 49 cm, 25 cm, and 1 cm above the bottom of the trap. Vertical black lines indicate when the dagger starts moving. PMT noise spikes can be seen as a result of dagger motion.

Daytime beam-off backgrounds, as well as beam-on position dependent backgrounds, are used to calculate height dependent factors. These dedicated beam-off backgrounds, seen in figure 4.27, spend 250 s at each height utilized in the UCN\(\tau\) production cycle. To avoid noise spikes, dedicated background runs calculations exclude times when the dagger is in motion. A height dependent correction for the background can be formed by assuming a functional form of the background:

\[
B (h) = f_h (h) B (h = 0). \tag{4.33}
\]
The height dependent factor $f_h(h)$ for each PMT threshold has been calculated for each PMT. The two PMTs do not behave the same, with different values of $f_h(h)$. The background can be divided into 4 different regions, as the height dependence of the rates shifts throughout the beam cycle.

The values for $f_h(h)$ can be measured for each dagger height across many different daytime background runs. For a given PMT and threshold, the counts at a given height can be summed, excluding the first 10 s to allow the dagger to cool. These counts can then be normalized to the bottom of the trap to determine the height dependent factors. These factors can be seen in figure 4.28

Using the daytime dedicated background runs to determine the behavior of the dagger during nighttime running assumes that the two types of runs are the same. This assumption cannot be adequately checked, as there are significantly fewer nighttime background runs when compared to daytime runs. Furthermore, the daytime background runs use a different pattern of motion than normal production. For more on the
potential daytime and nighttime difference and the implications for additional single PMT analysis uncertainties, see appendix B.

Figure 4.29: Height dependent factor $f_h(h)$ for each PMT threshold, separated by PMT as well as coincidence counting. This data is taken from the beginning of 2018. As the dagger raises, the single photon background rate increases while the coincidence background rate decreases. Note that for this particular background section, there is a 3σ discrepancy between high and low thresholds for PMT2.

Averaging the height dependence over all runs within a constant dagger condition can be used to determine the height dependent factors. The result of this is seen in figure 4.29. Single photon backgrounds typically increase as the dagger raises, while coincidence backgrounds decrease. Single photon backgrounds include dagger dark rate as well as real events that impart less energy than the multi-photon coincidence threshold. Geometric factors, such as PMT visibility of the trap, cause a greater percentage of changes in counts for single photons. Coincidence backgrounds are more likely caused by non-UCN real events. As such, coincidence backgrounds will be increased by a higher
ambient radioactive background. If UCN activate aluminum surfaces of the trap, then proximity to the trap leads to an increase in the background rate.

The contribution of position dependence to the overall background correction can be measured by calculating a lifetime with and without a position dependence correction. For a single photon analysis, calculating a lifetime without determining position dependence shifts the lifetime by $0.55 \pm 0.03$ s. As backgrounds have been suppressed in the coincidence case, turning on and off position dependent backgrounds only shifts the lifetime by $0.03 \pm 0.01$ s. An additional limiting case can be measured by using only nighttime beam-on runs to calculate the position dependent components of the background, rather than the daytime backgrounds presently used. The pattern of motion for daytime backgrounds differs from the nighttime runs, which could potentially affect the cooling and thus the raw height dependence. Daytime backgrounds would also be more susceptible to minor light leaks. During 2018, not enough data was taken for nighttime backgrounds to appropriately determine the shift in lifetime. For a single photon analysis, the variation between daytime and nighttime background runs for the height dependence can shift the background by $0.30 \pm 0.03$ s. Again, as the coincidence backgrounds are suppressed, the relevant shift for these is only $0.02 \pm 0.01$ s. This number can be used as the uncertainty due to position dependence.

4.5.4 Time Dependent Backgrounds

In addition to position dependent backgrounds, there is evidence for time dependence in single photon background events. This background rate differs between PMT1 and PMT2, and does not appear to be present in coincidence analysis. Multiple mechanisms to describe time dependent backgrounds have been proposed.

A handful of radioactive elements in the area could contribute to the time dependence in backgrounds. The vacuum jacket for the trap mainly consists of aluminum. When UCN enter the trap, they can potentially activate ambient aluminum to form $^{28}\text{Al}$,
Figure 4.30: Time dependence of background rates during summed long holding time runs. The two PMTs have been scaled such that the average rate in each is defined as unity. The residual takes the difference of these two scaled holding times.

which has a half-life of 134.7 s. Residual radioactive gases from further upstream in the source, such as $^{41}$Ar with a characteristic lifetime of 6560.4 s, could also enter the trap. Additionally, as UCN decay, $\beta$-decay products themselves could potentially interact with the dagger or PMTs, creating a minimal amount of light with a lifetime the same as $\tau_n$.

Time dependent backgrounds could also potentially come from changes in cooling of the dagger and PMTs. As the dagger moves up and down in the trap, the water lines keeping the PMTs at $\sim 5$ C extend and compress. The time taken for the dagger to reach equilibrium temperature with variable cooling geometries could also have an effect on the dark rate. This explanation could potentially account for the discrepancy between the two PMTs seen in figure 4.30.

Even without knowing the exact mechanism of time dependent backgrounds, the
time constants present in the backgrounds can still be fit using summed counts from the long holding time. These time constants must be found from runs with UCN in the trap. If a potential time dependent background source is caused by UCN activating materials, such as $^{28}\text{Al}$, dedicated background runs would not see a large time dependent effect. Instead, the background rate during the 1550 s holding time has been summed during consistent dagger conditions. This is then fit to a time dependent component of the background:

$$B\left(t'\right) = \alpha_1 e^{-t'/\tau_1} + \alpha_2 e^{-t'/\tau_2} + B\left(t' \to \infty\right). \quad (4.34)$$

The two time constants of equation (4.34), $\tau_1$ and $\tau_2$, with arbitrary scaling factors $\alpha_1$ and $\alpha_2$, provide an overall effective time constant averaged over various potential sources of background. This fitting occurs independently for each PMT and for coincidence data. The scaling factors for coincidence data, however, are consistent with zero time dependence. Combining equations (4.33) and (4.34) then provides a general background model:

$$B\left(h, t'\right) = f_h\left(h\right) \left(B\left(h = 0, t' \to \infty\right) + \alpha_1 e^{-t'/\tau_1} + \alpha_2 e^{-t'/\tau_2}\right). \quad (4.35)$$

The combined background model in equation (4.35) makes an assumption that the time-dependent background has no height dependence. In the event that the time dependence comes from changes in temperature due to cooling line geometry changes, this assumption might be insufficient. Not enough background data has been taken to independently extricate the height and time dependence.

Time-dependent backgrounds can have a non-negligible effect on the single PMT analysis lifetime, but do not change the coincidence lifetime. For single photons, the time dependent background can shift the background estimates by $\sim 1 \text{ Hz}$, or about 0.5%, depending on the specific background region model. The shift due to a time
dependence in the background estimate thus contributes 0.01 s ± 0.01 s, measured by calculating a lifetime with and without the time dependence for low-threshold combined PMTs. A similar calculation for the coincidence background is limited by uncertainty in the overall background rates, and thus the time dependent shift is below 0.01 s and has already been included in the background uncertainty.

4.5.5 Temperature Dependent Backgrounds

A single PMT dark rate varies as a function of temperature. Since the experimental area is not climate controlled, the ambient temperature potentially swings by ∼ 10 C between daytime and nighttime running. As described in section 3.3.3, the dagger has two PMTs kept at ∼ 5 C to reduce temperature-dependent noise. A water-ethanol mixture flows through cooling lines in order to keep the dagger at a roughly constant temperature. Nevertheless, small shifts in temperature lead to potential dark noise rate shifts in the dagger.

At the end of the 2018 data running, when PMT1 was replaced, thermocouples were placed inside the dagger assembly near the PMTs. This allows a measurement of the temperature in each PMT. The temperature dependence of background rates can be seen in figure 4.31. Fitting the temperature dependence to a straight line, indicates that a 1 K increase in temperature corresponds to a 10% increase in PMT dark noise. Temperatures in this dataset fluctuated around by around ∼ 1 K for PMT1, and a ∼ 3 K for PMT2. Additional background runs, taken while the dagger cooled, show deviation from a simple linear model. As only one subset of the data featured temperature dependence, background models did not use any temperature information for this analysis. Temperature dependence could be a potential avenue for improving background models during future running.
Figure 4.31: Temperature dependence in PMT1 and PMT2 for the end of 2018. The measured temperature for both PMTs has been plotted against the average background rate in each PMT for this dataset. Both PMTs show some evidence of correlation between the temperature and the rate.

4.6 NORMALIZATION

4.6.1 Overview

An accurate determination of the neutron lifetime requires absolute knowledge of the number of neutrons in the UCNτ trap. A spallation neutron source generates slightly different numbers of neutrons each time the beam hits the target. Small temperature fluctuations and minor inconsistencies in the proton beam spot position and size affect the efficiency of UCN production and transport out of the source. These run-to-run changes not only change the total number of UCN generated, but also change the energy spectrum of neutrons able to enter the guides. Prior to the hold, it is impossible to
directly measure the absolute number of UCN in UCN$_\tau$, as this would empty the trap. In order to determine the number of neutrons in the UCN$_\tau$ trap, normalization detectors must be used to reconstruct the UCN source’s production, while accounting for changes in the neutron energy spectrum. The goal of normalization is to find some method to relate a given run’s expected counts, $N_{\tau k}$, with that run’s various reconstructed monitor values $M_{i,k}$:

$$N_{\tau k} = \mathcal{F}(M_{0,k}, M_{1,k}, \ldots) .$$

(4.36)

Figure 4.32: Ratio of measured yields, $Y_s$, to the expected yields, $N_{\tau}$, generated using a single 20s holding time. A proper normalization scheme should remove any fluctuations in the source, and thus not have the characteristic “sawtooth” pattern of the raw counts. Whatever the choice of normalization methods, the normalized unload counts should form consistent groups, divided by holding times. The ratio between any two normalized holding times can then be used to form a lifetime.

Choosing the right form of $\mathcal{F}(M_0, M_1, \ldots)$ then provides a way to account for drifts
in the spectrum on a run-to-run basis. Various schema for determining \( F(M_0, M_1, \ldots) \) will be investigated in section 4.6.2 and section 4.6.3. Figure 4.32 shows one example normalization method, using a single 20 s holding time to determine \( F(M_k) \). This method will be discussed further in section 4.6.4. A different normalization scheme utilizing all the holding times will be described in section 4.6.5. These two normalization methods will be used as part of the overall lifetime calculation.

### 4.6.2 Normalization Model

As described in section 3.2.4, the various upstream monitors sample the spectrum coming out of the UCN source. The most naïve method of normalization would be to take the background subtracted unload counts and divide that by the counts out of a given monitor.

As seen in figure 4.33, a single monitor is not enough to fully describe the counting statistics of the trap. The distribution of UCN coming out of the source changes for two different normalization monitors, and discrete changes in the normalization monitors happen with various changes in PMT gains.

The spectrum coming out of the UCN source follows a power law distribution. UCN with energies above the guide system’s Fermi potential \( E_f \) are lost. These two competing physical properties give a total probability distribution of UCN energies in our system as:

\[
P(E) = \begin{cases} 
\alpha_1 E^x & E < E_f \\
\alpha_2 S(E) & E \geq E_f 
\end{cases}.
\]  

Equation (4.37) features arbitrary constants \( \alpha_{1,2} \) and an arbitrary loss function \( S(E) \).
Figure 4.33: Single monitor normalization during late 2018, dividing the reconstructed $Y$ counts by a single monitor. The upper plot divides the yield by the RHAC monitor, $M_H$, while the lower plot divides by the lower RH monitor, $M_L$. Discrete steps occur with both source MRFs and changes in PMT gains. Long term drifts in the upper monitor show the conditioning of the source as the beam hits the target, while these long term drifts are less prevalent in the lower monitor.
for overthreshold neutrons. As a simple model, define the tail density $\gamma_t$ as:

$$\gamma_t = \int_{E_f}^{\infty} a_2 S(E) \, dE.$$  (4.38)

Given an energy threshold $E_L$, the number of UCN counted can be easily algebraically divided into two energy regimes:

$$N_{\text{tot}} = N_{E<E_L} + N_{E\geq E_L}.$$  (4.39)

With an upper energy threshold $E_H > E_L$, where both are below $E_f$, the fraction $f_E$ of neutrons counted between the two energies, compared to all neutrons below energy $E_L$, is given by:

$$f_E = 1 - \left( \frac{E_L}{E_H} \right)^x.$$  (4.40)

Combining equations (4.37), (4.39) and (4.40), and subtracting off the tail part of the distribution defined in equation (4.38), the number of UCN in between two energies $E_L, E_H$ becomes:

$$N_{(E_L \leq E < E_H)} = \left[ \left(1 - \left( \frac{E_L}{E_H} \right)^x \right) \right] \left[ \frac{1}{\left(1 - \left( \frac{E_H}{E_f} \right)^x \right)} \right] \left( \frac{N_{(E \geq E_H)} - \gamma_t \cdot N_{\text{tot}}}{N_{\text{tot}} - \gamma_t} \right).$$  (4.41)

Each normalization monitor has an energy minimum, but can count anything above that with a roughly constant efficiency. In the case that one of our normalization monitors has an energy threshold at $E_H$ and in the limit $\gamma_t \to 0$, the expected number of
neutrons in the trap reduces to:

\[ N_T = \alpha_c M_H \left( 1 - \beta_s \frac{M_H}{M_L} \right) \]  \hspace{1cm} (4.42)

Efficiency and energy factors, which should be roughly consistent on a run-by-run basis, have been absorbed into two factors, a proportionality constant \( \alpha_c \) and a spectral coefficient \( \beta_s \). Although equation (4.42) makes some assumptions about the relative energies of normalization monitors, the scaling factors \( \alpha_c \) and \( \beta_s \) do not require the principal monitor to have a higher energy threshold than the spectral monitor, just that the two monitors sample different energy populations. For simplicity, the “main monitor,” the term in equation (4.42) will be further denoted as \( M_0 \), while the “spectral” term, which can be the ratio of monitors, will be denoted as \( M_s \). Since \( \alpha_c \) and \( \beta_s \) are arbitrary coefficients, changing the relative monitor energies will not change the form of equation (4.42).

UCN\( \tau \) ultimately attempts to measure the neutron lifetime. By fitting the normalization counts from equation (4.42) into equation (4.1), the number of neutrons unloaded for a given run becomes:

\[ Y(t') = (\alpha_c M_0 + \beta_s M_s) e^{-t'_0/\tau_n} e^{-t'/\tau_n} \]  \hspace{1cm} (4.43)

Again, since \( \alpha_c \) and \( \beta_s \) are arbitrary constants, an initial time multiplicative factor \( e^{-t'_0/\tau_n} \) can scale the normalization monitors to any desired initial time. However, the actual determination of these parameters in equation (4.43), in particular when dealing with datasets of varying statistics, can have a statistical bias due to whichever model of counting is chosen.
4.6.3 Investigating Model Choices

As an investigation of the effectiveness of various possible normalization models, the $\chi^2$ of various fitting models was calculated. Unload data from 20 s holding times inside of a RB was fit to the raw monitor counts, with $\alpha_c$ and $\beta_s$ as free parameters. The $\chi^2$, assuming a Gaussian fluctuation in the normalized yields within each RB was calculated for the entire year’s worth of data as well as the maximum $\chi^2$ for each individual section, assuming purely statistical uncertainties.

<table>
<thead>
<tr>
<th>Function</th>
<th>2017 $\chi^2$/NDF</th>
<th>2018 $\chi^2$/NDF</th>
<th>2018 Max $\chi^2$/NDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_T = \alpha_c M_L$</td>
<td>37.871</td>
<td>3.880</td>
<td>2.564</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_H$</td>
<td>414.023</td>
<td>64.302</td>
<td>80.287</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_L + \beta_s M_H$</td>
<td>6.994</td>
<td>4.585</td>
<td>3.893</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_L + \beta_s \frac{M_L^2}{M_H^2}$</td>
<td>6.695</td>
<td>4.511</td>
<td>3.899</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_H + \beta_s \frac{M_L^2}{M_H}$</td>
<td>8.449</td>
<td>5.082</td>
<td>5.420</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_L + \beta_s \frac{M_H^2}{M_L}$</td>
<td>7.160</td>
<td>4.633</td>
<td>3.901</td>
</tr>
<tr>
<td>$N_T = \alpha_c M_H + \beta_s \frac{M_H^2}{M_L}$</td>
<td>6.574</td>
<td>4.473</td>
<td>4.039</td>
</tr>
</tbody>
</table>

Table 4.2: $\chi^2$/NDF values for various potential normalization schemes using either a low monitor (GV in 2017 or RH in 2018) or high monitor (SP in 2017 or RHAC in 2018) as the principal normalization monitor

A summary of these results can be found in table 4.2. Data taken in 2017 uses GV as low monitor $M_L$ and SP as high monitor $M_H$, while data taken in 2018 uses RH as low monitor $M_L$ and RHAC as high monitor $M_H$. The high monitor $M_H$ exhibits significantly non-Gaussian behavior as a linear model. While normalization schemes in 2018 are relatively consistent and model-independent, using the low monitor $M_L$ as the principal normalization monitor better captures the normalization in 2017. The presence of the Roundhouse as a buffer volume between the source and the trap leads to the $\chi^2$ improvement between 2017 and 2018. Any of the physically derived normalization models provide a consistent $\chi^2$/NDF of $\sim 4.5$ in 2017 and $\sim 2.2$ in 2018, while a simple linear model does not provide adequate modeling of the source behavior in 2017. In
order to adequately describe both 2017 and 2018 and account for the increased stability of the lower monitor as a primary normalization model, the “best” normalization function is chosen to be:

$$\mathcal{F}(M_{0,k}, M_{1,k}, \ldots) = \alpha_c M_L \left(1 - \beta_s \frac{M_L}{M_H}\right).$$  \hspace{2cm} (4.44)

In 2018, both the SP and RHAC monitors can be used as the upper monitor, while the GV and RH monitors both can be the lower monitor. A quantitative test of the normalization monitors, then, would be to compare the lifetimes calculated with the GV and SP monitors with the lifetimes using the two RH monitors. In 2018, which itself has a paired statistical uncertainty of ±0.61 s, swapping out just the RHAC for the SP monitor raises the calculated lifetime by 0.03 ± 0.06 s, while swapping out both monitors raises the lifetime by a total 0.13 ± 0.11 s. The increase in statistical uncertainty is primarily due to a lack of statistics in the SP monitor; the RHAC counts the majority of the overthreshold UCN. Additionally, the addition of the RH smooths irregularities in the beam, increasing the effectiveness of the normalization scheme, which the GV does not encompass.

An additional monitor, sampling a different area of the section, could be the DS monitor, which would indicate inconsistencies in the TD and CD positions. A similar procedure to the one described above was used to isolate the added spectral component of the downstream monitor. Equation (4.44) can be perturbed by adding either $\gamma M_{DS}$, or potentially in analogy to the previously described spectral terms, $\gamma \frac{M_{DS}^2}{M_{DS}}$.

In this case, we have introduced an additional term that describes the source output and the trappable neutrons. If the normalization model were insufficient, the $\chi^2$ for a higher order term would be consistently lower. As seen in table 4.3, the DS monitor provides only a minimal amount of extra information, and does not bring the $\chi^2$/NDF all the way to one. Based on this, it can be concluded that the TD position is consistent and that two monitors adequately describe the output of the source.
\[ N_\tau = \alpha_c M_L + \beta_s M_{hi}^2 \]

\[ N_\tau = \alpha_c M_L + \beta_s M_{hi}^2 + \gamma M_{DS} \]

<table>
<thead>
<tr>
<th>RB</th>
<th>$\chi^2$/NDF</th>
<th>$\chi^2$/NDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>9767</td>
<td>3.238</td>
<td>3.145</td>
</tr>
<tr>
<td>9960</td>
<td>3.960</td>
<td>3.802</td>
</tr>
<tr>
<td>10936</td>
<td>0.578</td>
<td>0.467</td>
</tr>
<tr>
<td>10988</td>
<td>4.387</td>
<td>4.169</td>
</tr>
<tr>
<td>11085</td>
<td>1.937</td>
<td>1.917</td>
</tr>
<tr>
<td>11669</td>
<td>1.872</td>
<td>1.838</td>
</tr>
<tr>
<td>12516</td>
<td>1.857</td>
<td>1.836</td>
</tr>
<tr>
<td>13209</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>13307</td>
<td>1.933</td>
<td>1.930</td>
</tr>
</tbody>
</table>

Table 4.3: $\chi^2$/NDF values in 2018 RBs with and without the DS monitor as an additional normalization monitor. The addition of the DS does not have a significant overall impact on the $\chi^2$/NDF, which indicates that the TD does not provide an explanation for the any non-Gaussian behavior. The run segment between 13209 and 13307 does not have enough 20 s holding times to calculate a $\chi^2$.

4.6.4 Single Holding Time Normalization

The most straightforward method of calculating spectral coefficients $\alpha_c$ and $\beta_s$ involves choosing a single fixed holding time. Then the normalization proceeds through two steps, first finding the expected counts $N_\tau$ and then combining these to extract the lifetime $\tau_n$. Utilizing multiple holding times would add a dependence of $\alpha_c$ and $\beta_s$ on $\tau_n$. As will be shown in section 4.6.5 this is possible, though limited by heavy computing requirements. Thus, for systematic studies as well as paired lifetime studies, a single holding time normalization is very useful.

Determination of $\alpha_c$ and $\beta_s$ spectral parameters proceeds through the process of non-linear least-squares fitting to unload counts of a single nominal holding time, $t_N$. In between two RBs, monitor counts should have the same behavior. For such a normalization subsection, Scipy’s “curve_fit” function fits all runs with holding time $t = t_N$ with reconstructed yields $Y_k = U_k - B_k$ by minimizing:

\[
\chi^2 = \sum_k \left[ \frac{(U_k - B_k) - (\alpha_c M_{0k} + \beta_s M_{sk})}{\sigma_{U_k,B_k}} \right]^2.
\] (4.45)
Scipy returns optimal values for $\alpha_c$ and $\beta_s$, as well as the associated covariance matrix. Runs with a holding time $t \neq t_N$ then are assigned values of $\alpha_c$ and $\beta_s$ based on their nearest normalization neighbor in the same normalization subsection. Since the least-squares fit occurs during a single holding time, the normalization time $t_N$ appears in $\alpha_c$ and $\beta_s$ as a factor of $e^{-t_N/\tau_n}$.

Given a $2 \times 2$ covariance matrix with elements indicated as $C_{c1,c2}$ and expected number of neutrons given by equation (4.43), the uncertainty on the expected number of neutrons is:

$$
\delta(N_{\tau}) = \sqrt{N_{\tau} + \delta(M_0, M_s)^2 + M_0^2 C_{1,1} + M_s^2 C_{2,2} + 2M_0 M_s C_{1,2}}.
$$

In the uncertainty calculation, the $\sqrt{N_{\tau}}$ term indicates a Poisson fluctuation in the normalization counts, $\delta(M_0, M_s)$ is the uncertainty in the monitor counts themselves, and the remaining terms are the uncertainty due to the least-squares fit.

A least-squares fit fails when attempting to fit yields and monitor counts to sparse matrices. Thus, in run segments with too few normalization runs, a handful of edge cases provide an approximation for the normalization coefficients. Since $C$ is a $2 \times 2$ matrix in this normalization model, normalization subsets with one or two runs cannot proceed through this least-squares fitting procedure. With exactly two valid normalization runs, $\alpha_c$ and $\beta_s$ are solved for exactly, while with just one valid normalization run $\beta_s = 0$ and $\alpha_c$ is solved for exactly. In both of these cases, the uncertainty on the expected number of neutrons is poorly defined. Such sparse normalization regions are typically excluded from lifetime analyses and only used for systematic tests.

The spread in normalized counts due to a single holding time normalization can be seen in figure 4.34. In particular, the distribution of both the distribution of $Y_k$ for short and long holding times can be well-described by a Gaussian distribution.
Figure 4.34: Histogram of the ratio of measured yields, $Y$, to the expected yields, $N_\tau$. Histograms have been centered such that the mean of a given holding time is at 0. The Gaussian has been chosen based only on the standard deviation of the normalization counts. As the normalization model only uses the 20 s holding time for normalization, the agreement between short and long holding times is very good.

4.6.5 Global Markov Chain Monte Carlo Normalization

The single holding time normalization described in section 4.6.4 does not optimally use statistics, as only one holding time determines $\alpha_c$ and $\beta_s$. A more statistically robust model would incorporate the scaling factor $e^{-t'/\tau_n}$, as seen in equation (4.42), in the calculation of the normalization coefficients. In this case, a variation in $\tau_n$ would have some correlation with the normalization coefficients $\alpha_c$ and $\beta_s$. As this requires a more iterative process, a least-squares fitting is no longer sufficient. Instead, a method of simultaneous lifetime and normalization fitting must use a likelihood fitting of our observables.

First, an overall likelihood function should relate the various monitor counts and
yields across all our runs. For a given run, we can determine the reconstructed unload counts $U_k$ and the expected background counts $B_k$. The background counts within a given region should follow some distribution close to, but not exactly, Poisson. In this case, within a given run segment one can define an average background rate $\bar{B}$ and marginalize over this parameter. Assuming a Poisson distribution of unload counts and an expected normalization value $N_\tau$ from equation (4.42), the likelihood for counting the values that we see on a single run becomes:

$$L_{(U,k)} \left( \tau_n, U_k, N_{\tau k}, B \right) = \frac{\left( N_{\tau k} e^{-t'/\tau_n + \bar{B}} \right)^{U_k} e^{-\left( N_{\tau k} e^{-t'/\tau_n + \bar{B}} \right)}}{U_k!}.$$  \hspace{1cm} (4.47)

In order to appropriately marginalize over the average background $\bar{B}$, the background is also treated following a Poisson distribution:

$$L_{(B,k)} \left( \bar{B}; B_k \right) = \frac{\left( B \right)^{B_k} e^{-\bar{B}}}{B_k!}.$$  \hspace{1cm} (4.48)

While these models work for coincidence counting methods, for single photon analysis, the counted distribution is not Poisson. Instead, the distribution of photons from a neutron event in itself has some distribution which must also be marginalized over. If the number of photons counted by a given UCN event roughly follows a Gaussian distribution, then the marginalized background value $\bar{B}$ and the unload value $U_k$ can be scaled by an efficiency factor $\epsilon_{Tk}$. This efficiency factor can be modeled through the introduction of an average scaling factor, $\bar{\epsilon}$, in the likelihood analysis. Assuming that this factor is roughly Gaussian distributed gives:

$$L_{(\epsilon_{T,k})} \left( \bar{\epsilon}; \epsilon_{Tk}, U_k, B_k \right) \sim e^{-\frac{(\epsilon_{Tk}-\bar{\epsilon})^2(U_k-B_k)}{\epsilon_{Tk}^2(e_{Tk+1})}}.$$  \hspace{1cm} (4.49)

The total likelihood for the dataset can be found by multiplying equation (4.47),
equation (4.48), and equation (4.49) over all runs:

\[
\mathcal{L} (\tau_n) = \prod_{k}^{N_k} \mathcal{L}_{(U,k)} (\tau_n; U_k, N_{\tau_k}, B) \times \mathcal{L}_{(B,k)} (\bar{B}; B_k) \times \mathcal{L}_{(\epsilon_T,k)} (\bar{\epsilon}; \epsilon_{Tk}, U_k, B_k). \tag{4.50}
\]

The value of \(\tau_n\) and the marginalizable normalization parameters can then be found at the maximum value of this \(\mathcal{L} (\tau_n)\). For a coincidence analysis, equation (4.49) is fixed as a constant, since in the case where \(U\) has been appropriately corrected for Rate Dependent Effects (RDE)s, each UCN produces one \(U\) count. One potential adjustment to this likelihood model thus would separately deal with RDEs. However, since the rate dependent effects are heavily correlated within a given run but not correlated with other runs, this would introduce a new marginalizable parameter for each run. As a result, the additional uncertainty due to RDEs must be accounted for via other means. A complete likelihood model would additionally include a likelihood term involving weighting of reconstructed monitor counts in equation (4.50). The monitor counts \(M_0\) and \(M_s\) do not follow an easily computable distribution, as each run features potentially different source conditions and thus different monitor distributions. Nevertheless, the rates in the normalization monitors are an order of magnitude higher than the rates in unload detectors, and thus contribute a negligible component towards the overall uncertainty.

Calculation of the likelihood function proceeds by taking the log likelihood of equation (4.50). Maximization of a many-dimensional function is a hard problem, and the product of observables from every production run takes too much processing power to be realistic. The logarithm of the likelihood, assuming finite counts, has the same maximum values as the “real” likelihood. Since the logarithm of products becomes the sum of logarithms, a log likelihood function, \(\mathcal{M}\) reduces to a large number of sums. The total
calculable probability then becomes:

\[ M (\tau_n) = \sum_k^{N_R} \log L_{(U,k)} (\tau_n; U_k, N_{\tau_k}, B) + \log L_{(B,k)} (B; B_k) + \log L_{(\epsilon,\epsilon_{T,k}, U_k, B_k)} (\epsilon, \epsilon_{T,k}, U_k, B_k). \]  

(4.51)

Marginalizing over this multiparameter distribution for the many independent normalization subsections proceeds through the Markov Chain Monte Carlo (MCMC) Python package “emcee”[69]. The MCMC algorithm samples areas of parameter space in order to marginalize over parameters. The output of this algorithm over a subset of the data appears in figure 4.35. Regions with a higher likelihood are more heavily sampled, and thus histogramming independent MCMC events allows us to find the most likely values for marginalizable parameters. For a given distribution, the uncertainties on \( \tau_n \), as well as whichever other normalization parameters we wish to determine, come from the one sigma bands of the MCMC histograms.

4.7 PAIRED AND GLOBAL LIFETIME CALCULATION

4.7.1 Overview

The lifetime in UCN\( \tau \) comes from fitting a decay exponential over a large number of individual runs:

\[ \langle N (t') \rangle = N (t'_0 = 0) e^{t'/\tau_n}. \]  

(4.52)

Previous sections have discussed how to generate observables for each run, in particular methods for finding \( N (t') \) and \( N (0) \) for each run. Given the large number, \(~ 5000,\) of possible production runs, reconstruction of the lifetime can be complicated by statistical imprecision and variations in relative uncertainties between monitor counts. Minor shifts in statistical and systematic precision between individual runs can lead to non-trivial lifetime biases, and so care must be taken with lifetime reconstruction.
Figure 4.35: Corner plot showing the correlations between $M$ parameters, with the MCMC taking data from coincidence events for the end of 2017. Normalization parameters $\alpha_c$ and $\beta_s$ have heavy correlation, while samples of $\tau_n$ are not as correlated. For legibility, the associated MCMC results for the background value have not been plotted.

Different methods of calculating the lifetime can be complementary. The two primary methods of lifetime calculation are global and paired. A “global lifetime” analysis utilizes all the data, which maximizes statistics, but would be more subject any to systematic biases in normalization or background modeling. In a “paired analysis,” each run with a short holding time is combined with a nearby run with a long holding time. Then, the decay exponential can be rewritten as with combination of run holding times $t_S, t_L$ and the normalized yields $Y_S, Y_L$, and becomes:

$$
\tau_n = \frac{t_L - t_S}{\ln \frac{Y_S}{Y_L}}. \quad (4.53)
$$
The paired analysis, by comparing runs with similar run states, then reduces the effect of any systematic uncertainties in normalization or background reconstruction. The global and paired lifetime methods can be utilized for any type of event reconstruction. Both the single photon counting mode, section 4.4.3, and the coincidence counting mode can be used with either method. Backgrounds, discussed in section 4.5, have increased the difficulty of an analysis using a single photon counting mode. Nevertheless, for a single photon counting mode, the lifetime can be calculated with either PMT1, PMT2, or both. Any of these methods of measuring the lifetime can be used with either high or low PMT thresholds. This leads to a total of 16 combinations that can be used to calculate the lifetime, each of which provides complementary information.

4.7.2 Global Lifetime

The global lifetime, as previously described in section 4.6.5, incorporates the lifetime as a free parameter during the normalization process. As the global lifetime has been explicitly tied to the normalization, the actual method by which the lifetime is calculated will not be revisited here. The MCMC software records the lifetime, normalization, and the likelihood for each event sample. Thus, the global lifetime, $\tau_n$, comes from the MCMC sample with the highest likelihood. The $1\sigma$ uncertainty on the lifetime can be found by calculating the 16th percentile and the 84th percentile of the sampled MCMC events. In general these percentiles will not be perfectly symmetric, as the samplings start from a slightly offset initial guess. To accommodate this, the quoted statistical uncertainty has been chosen as the percentile further from the maximum likelihood point.

Due to computing limits, the global lifetime must be calculated independently for 6 separate sections of data, with these 6 independent lifetimes then using a weighted mean to reconstruct the total lifetime value. Since each independent global lifetime calculation utilizes a large amount of data, the statistical bias of a weighted mean of these values is
negligible.

4.7.3 Paired Lifetime

Run pairing combines a single short holding time run with a single long holding time run to find a lifetime. In order to extricate the normalization from the lifetime, a single holding time normalization, described in section 4.6.4, must be used. Calculation of the lifetime utilizes all available 20 s runs within a given break. Then, nearby short runs, with a holding time < 1000 s can be paired with a nearby long run, with holding time > 1000 s.

**Algorithm 5** Pairing Algorithm

1: Run Numbers = S, L
2: Holding Times = tS, tL
3: Normalized Yields = YS, YL
4: Expected Neutrons = NS, NL
5: for j = 1; j < 16; j++ do
6: if |S − L| ≤ j and S, L in same run break then
7: if |S − (L + 1)| < |S − L| then
8: continue
9: else if |S − (L + 1)| = |S − L| then
10: if 1 − NS/N(L+1) > 1 − NS/NL then
11: continue
12: end if
13: end if
14: if 9/10 < NS/NL < 10/9 then
15: S, L are paired! Remove these from the set and increment S, L
16: end if
17: end if
18: end for

The pairing algorithm described in algorithm 5 simultaneously chooses short and long runs with a minimal temporal displacement, prioritizing similar predicted unloads. Short and long runs must have an expected number of counts within 90% of each other and must be within 16 runs of one another. Runs with similar N are prioritized over temporally closer runs, as this helps reduce potential biases due to normalization. With minimal quality cuts, only removing runs with poor tagbits, the pairing algorithm gen-
erates 2207 pairs out of 5035 runs. This corresponds to a pairing efficiency of 87.7\% of normalizable runs being used for the lifetime measurement. The reduced run efficiency inflates the uncertainties determined from a paired lifetime measurement when compared with a global lifetime measurement. In reality, the pairing efficiency will vary from this value, as quality cuts reduce the spread of the normalization but could also thin the number of available runs. Non-paired runs still provide data for normalization and background estimates.

A potential statistical bias issue appears when averaging these unloads to form a real lifetime. When dealing with a large number of counts, averaging \( \tau_n \) from all of our different runs will provide an unbiased estimate of the lifetime. However, in the case where there is a statistical distribution in \( Y_S \) and \( Y_L \), averaging the lifetimes together biases the averaged lifetime. Given a statistical spread of yields:

\[
\frac{\langle Y_S \rangle}{\langle Y_L \rangle} = \frac{\langle Y_S \rangle}{\langle Y_L \rangle} + \delta(Y_S) \frac{Y_S^2}{Y_L^2} + \delta(Y_L) + \ldots. \tag{4.54}
\]

The value determined with a maximum statistical reach would be the term on the left in equation (4.54), and thus this term provides the most unbiased value. The deviation from the unbiased value can be found with a Taylor expansion around \( Y_S \) and \( Y_L \). Defining \( r \equiv \frac{Y_S}{Y_L} \) and expanding around the mean values:

\[
\frac{\langle Y_S \rangle}{\langle Y_L \rangle} = \frac{\langle Y_S \rangle}{\langle Y_L \rangle} - \delta(Y_L) \left( Y_S \frac{\delta(Y_L)}{Y_L} + \delta(Y_S) \right) + \ldots. \tag{4.55}
\]

This gives a correction to the ratio, \( r = \frac{\langle Y_S \rangle}{\langle Y_L \rangle} \). The weighted mean value of the ratio in the denominator of equation (4.53) can thus be corrected using equation (4.55). A similar Taylor expansion can be done to correct for the statistical bias inherent to equation (4.53), by expanding the average \( \langle \tau_n \rangle \) around the mean ratio \( \langle r \rangle \):

\[
\langle \tau_n \rangle = \frac{(t_L - t_S)}{\ln \langle r \rangle} + \frac{1}{2} \frac{(t_L - t_S)(2 + \ln r)}{r^2 \ln^3 r} \delta(r)^2 + \ldots. \tag{4.56}
\]
These two biases can be calculated directly, in order to compensate for the statistical bias in the paired lifetime. A weighted average, using the uncertainty for yields and normalizations previously calculated, can then be taken to combine the lifetimes. This allows a more precise evaluation of a paired lifetime using the measured values in $Y_S$ and $Y_L$, along with their associated uncertainties.

![Distribution of paired lifetimes](image)

Figure 4.36: Histogram of the 2017-2018 dataset’s 1984 paired, blinded, lifetimes, combined across all running conditions and normalized such that the integral of the distribution is 1. The colors of the stacked histogram indicate the length of the short holding times: red for 20 s; yellow for 50 s; green for 100 s; and blue for 200 s. There is not a significant difference between the various short holding times, and thus the $\chi^2$ for each possible pairing time distribution has not been calculated. The overall paired lifetime distribution can be well described ($\chi^2/NDF = 0.953$) by a Gaussian with $\mu = \tau_n$ and $\sigma = \delta(\tau_n) \sqrt{N_{\text{pairs}}}$, indicating a successful normalization.

The spread of paired lifetimes can be seen in figure 4.36. In the case of significant statistical bias or a poor normalization, the distribution would have a poor $\chi^2$, and the median and mean lifetime values would potentially differ. This is not the case, as the
combined dataset has a total $\chi^2 = 1891.782$, corresponding to a $\chi^2/NDF = 0.954$. For a low-threshold coincidence analysis, a paired lifetime analysis gives a total blinded lifetime of $\tau_n = 887.66 \pm 0.31$ s.

4.8 SUMMARY OF RESULTS

The previous sections have illustrated various methods by which UCN$\tau$ determinates the lifetime. This analysis reports a statistical lifetime of $\tau_n = 887.66 \pm 0.31$ s for a paired analysis and $\tau_n = 887.64 \pm 0.26$ s for a global analysis.

<table>
<thead>
<tr>
<th>Counting</th>
<th>Threshold</th>
<th>Method</th>
<th>$\tau_n$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coinc.</td>
<td>Low</td>
<td>Paired</td>
<td>887.66 ± 0.31*</td>
</tr>
<tr>
<td>Coinc.</td>
<td>High</td>
<td>Paired</td>
<td>887.68 ± 0.32</td>
</tr>
<tr>
<td>Coinc.</td>
<td>Low</td>
<td>Global</td>
<td>887.64 ± 0.26*</td>
</tr>
<tr>
<td>Coinc.</td>
<td>High</td>
<td>Global</td>
<td>887.70 ± 0.27</td>
</tr>
<tr>
<td>PMT1</td>
<td>Low</td>
<td>Paired</td>
<td>884.71 ± 0.48</td>
</tr>
<tr>
<td>PMT1</td>
<td>High</td>
<td>Paired</td>
<td>884.91 ± 0.49</td>
</tr>
<tr>
<td>PMT1</td>
<td>Low</td>
<td>Global</td>
<td>884.69 ± 0.37</td>
</tr>
<tr>
<td>PMT1</td>
<td>High</td>
<td>Global</td>
<td>884.89 ± 0.39</td>
</tr>
<tr>
<td>PMT2</td>
<td>Low</td>
<td>Paired</td>
<td>887.82 ± 0.43</td>
</tr>
<tr>
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<td>High</td>
<td>Paired</td>
<td>887.81 ± 0.44</td>
</tr>
<tr>
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<td>Global</td>
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<tr>
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<td>Paired</td>
<td>886.47 ± 0.33</td>
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</tr>
<tr>
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<td>Global</td>
<td>886.35 ± 0.39</td>
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</tbody>
</table>

Table 4.4: Summary of blinded Neutron Lifetimes, with statistical uncertainties. The lifetimes marked with (*) are the reported numbers. Singles lifetimes use background estimates summed over runbreaks to compare between paired and global.

A full list of the possible analysis combinations, varying methods of lifetime calculation as well as which dagger are used for analysis, can be seen in table 4.4. Due to limitations in the understanding of backgrounds in the single photon analysis method, the individual PMT methods are not used for this analysis. Nevertheless, the combined PMT1 and PMT2 lifetimes are consistent with the total coincidence analyses. Similarly,
the high threshold for PMT counting, although consistent with the low threshold for any combination of PMTs and counting methods, contains potential gain shifts, and thus is not used for the lifetime calculation. As the global analysis can use more of the data than the paired method, the reported uncertainty is lower.

<table>
<thead>
<tr>
<th>Shift</th>
<th>Correction (s)</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMT1 Unweighted Monitors</td>
<td>−0.37 ± 0.02</td>
<td>4.2.4</td>
</tr>
<tr>
<td>PMT2 Unweighted Monitors</td>
<td>−0.38 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>PMT1 + PMT2 Unweighted Monitors</td>
<td>−0.47 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>Coinc. Unweighted Monitors</td>
<td>−0.45 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Singles RDE</td>
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<td>4.4.6</td>
</tr>
<tr>
<td>Coinc. RDE</td>
<td>−1.16 ± 0.03</td>
<td>4.4.7</td>
</tr>
<tr>
<td>PMT1 Individual vs. Average Bkg.</td>
<td>−0.64 ± 0.04</td>
<td>4.5.2</td>
</tr>
<tr>
<td>PMT2 Individual vs. Average Bkg.</td>
<td>−1.11 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>PMT1 + PMT2 Individual vs. Average Bkg.</td>
<td>−0.76 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Coinc. Individual vs. Average Bkg.</td>
<td>−0.08 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>PMT1 Day vs. Night</td>
<td>0.33 ± 0.05</td>
<td>4.5.3</td>
</tr>
<tr>
<td>PMT2 Day vs. Night</td>
<td>0.23 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>PMT1 + PMT2 Day vs. Night</td>
<td>0.30 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Coinc. Day vs. Night</td>
<td>0.02 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>PMT1 Position Dep. Background</td>
<td>−0.63 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>PMT2 Position Dep. Background</td>
<td>−0.46 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>PMT1 + PMT2 Position Dep. Background</td>
<td>−0.55 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Coinc. Position Dep. Background</td>
<td>−0.03 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>PMT1 Time Dep. Background</td>
<td>−0.01 ± 0.02</td>
<td>4.5.4</td>
</tr>
<tr>
<td>PMT2 Time Dep. Background</td>
<td>−0.01 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>PMT1 + PMT2 Time Dep. Background</td>
<td>−0.01 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>RHAC vs. SP (2018)</td>
<td>0.03 ± 0.06</td>
<td>4.6.3</td>
</tr>
<tr>
<td>RH + RHAC vs. GV + SP (2018)</td>
<td>0.13 ± 0.11</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Summary of systematic shifts accounted for in the statistical uncertainty. These contributions have been calculated by subtracting the uncorrected lifetimes from the corrected, real lifetime. Uncertainties on these quantities are only due to the associated increase in statistical uncertainty.

A list of “systematic” contributions to the statistical uncertainty have been enumerated in table 4.5. The systematic corrections listed here, in contrast with the systematic sources of loss which will be described in detail in chapter 6, should not be thought of as additional sources of uncertainty. Rather, the shifts illustrated here should demonstrate the potential effects of choices made by the analyzers that must be investigated.
and understood.
5.1 OVERVIEW

Measurement of the neutron lifetime using the UCN\(_{\tau}\) trap must have a good understanding of any sources of loss. However, although the many detectors in the trap and the monitors outside it provide some information on these loss mechanisms, reaching the UCN\(_{\tau}\) goal precision of 0.2 s requires detailed Monte Carlo (MC) methods. There is no way to measure a neutron without destroying it, and this complicates the investigation of systematic measurements. Each UCN event seen by a detector provides some information. Only the time at which that neutron interacts with the detector can be known for certain, as the detection of individual UCN is a quantum process with a finite probability. As a result, the energy and trajectory of a single UCN cannot reliably be determined using the dagger or cleaner detectors alone. A UCN lost for any reason during storage provides no information whatsoever to the analyzers. Thus, systematic mechanisms of loss can only be determined by investigating the averaged properties of UCN, principally through measuring differences between short and long holding times. For a 20 s holding time paired with a 1550 s holding time, the loss of one UCN out of 10,000 due to a non-\(\beta\)-decay mechanism would lead to a systematic shift in \(\tau_\text{n}\) of 0.29 s. A more in-depth description of possible sources of loss will be described in chapter 6. In order to claim knowledge of a systematic effect with uncertainty below the statistical uncertainty, we must be able to account for the movement of essentially every UCN, especially those close to the trapping potential.

A shift in lifetime can be determined by calculating the lifetime for a short/long pair with known numbers of neutrons. Using the MC method, such a lifetime shift can be
written as:

$$\Delta(\tau_n) = \tau_{MC} - \frac{t_{L,MC} - t_{S,MC}}{\ln \frac{Y_{L,MC}}{Y_{S,MC}}}$$

(5.1)

where $\tau_n$ is the lifetime put into the model and $\tau_{MC}$ is the extracted lifetime using the MC simulated yields $Y_{S/L,MC}$ and mean arrival times $t_{S/L,MC}$. This procedure can be repeated until the $\Delta(\tau_n)$ is known to the desired precision, where:

$$\delta \tau_{MC} = \frac{\sum_i^N \Delta(\tau_n)_i - \Delta(\tau_n)}{\sqrt{N}}.$$  

(5.2)

Here equation (5.2) is just the typical standard deviation of the mean. The number of simulations $N$ can be chosen to reach a $\delta \tau_{MC}$ of any arbitrarily low precision, such as $\delta \tau_{MC} \leq 0.01s$.

The UCN$\tau$ experiment has been complemented with a suite of various MC simulations as a tool to describe neutrons in the trap and to predict their associated losses. These simulations provide high statistics studies of various potential sources of loss and help to understand the neutrons lifetime to an even higher precision. Two different simulations have been developed. The first class of simulations involves tracing the trajectories of UCN in the trap. These provide knowledge of the motion of UCN within the trap, allowing investigations of changes in the phasespace distribution between short and long holding times and the resulting change in the efficiencies of spectral cleaning and detection. An overview of the trajectory simulations will be provided in section 5.2. The second class of simulations simulates the detector response for varying rates of UCN. These benchmark the coincidence reconstruction algorithms, and provide insights for improving the UCN$\tau$ detectors. More information about the coincidence detector simulations will be found in section 5.3. The combination of MC methods, in conjunction with data-driven systematic tests, provide the necessary tools towards future advances of UCN$\tau$ and measurements of the neutron lifetime.
5.2 TRAJECTORY SIMULATIONS

5.2.1 Overview

Neutrons travelling in the trap obey classical equations of motion. Since a neutron, in the classical limit, can be treated as a point particle with a magnetic dipole and a mass, their motion through a known magnetic field can be directly described by electrostatics. In order to describe the bulk distribution of neutrons and their changes in distribution between short and long holds, a series of trajectory simulations ran on Indiana University (IU)’s Big Red 3 (BR3) supercomputer. These simulations generate \( O(10^6) \) neutron trajectories for the explicit purpose of being able to separate short and long holding time differences, giving predictions on potential sources of loss, and checking the geometrical acceptance of our detectors.

The trajectory simulations can run in two modes. A “tuning” mode defines a handful of parameters which can vary. Simulated UCN trajectories record the first 50 hits upon detectors of interest. In post-processing, these hits are then reweighted by the means of \( \chi^2 \) minimization to compare against actual data taken by the experiment, in order to optimize the trap parameters. Once the simulations have been tuned, a “production” mode can generate neutrons using the model with the best fit parameters. This speeds up the simulation and allows simulated sources of loss to be added. For a short-long pair with holding times \( t_S \) and \( t_L \), an additional source of loss can be introduced and simulated. After running the simulation and counting \( Y_S \) and \( Y_L \) UCN, the yields can be scaled by a MC lifetime, \( \tau_{MC} = 880 \) s. This would shift the lifetime by:

\[
\Delta(\tau_n) = \tau_{MC} - \frac{t_L - t_S}{\ln \left( \frac{Y_S}{Y_L \times e^{-\left(\frac{t_L - t_S}{\tau_{MC}}\right)}} \right)}
\]  

Typically, 12 hours of running on BR3 allows tracking \( \sim 2 \) million UCN for both 20 s and 1550 s holding times, depending on various factors such as the number of available
detectors and the complexity of the fields utilized. The statistical uncertainty on such a measurement can be found through normal Gaussian error propagation, which reaches a precision $\sim 1 \times 10^{-4}$ s. If a simulated source of loss is worse for certain regions of phasespace, the overall UCN loss rate will be very sensitive to small variations in the initial distribution.

5.2.2 Trap Fields Model

Trajectory simulations of trapped UCN starts with an analytic model to describe the trap’s geometry and magnetic fields. UCN accelerate due to both magnetic and gravitational fields; given a constant gravitational field, a Cartesian coordinate $(\hat{x}, \hat{y}, \hat{z})$ is used. The UCN$_\tau$ trap consists of two toroidal segments forming a curved surface. These two toroids have a major radius, $R$, and a minor radius, $r$, connected along the $x = 0$ plane. On either side of this plane, one of these two radii is 1.0 m, while the other is 0.5 m. This introduces an asymmetry required to mix trajectories. A continuous analytic form for the surface of the trap can be written as:

$$
R(x) = \frac{1}{2} + \frac{1}{2(1 + e^{-\kappa x})},
$$

$$
r(x) = 1 - \frac{1}{2(1 + e^{-\kappa x})}.
$$

In order to maintain smooth forms of $R(x)$ and $r(x)$, equation (5.4) has introduced an arbitrary scaling constant, $\kappa = 1000$, which provides a $\sim 5$ mm region of crossover to avoid a discontinuity at $x = 0$. Note that $R(x) + r(x) = 1.5$ m for all values of $x$. The origin of our Cartesian axes is placed at the center of the toroidal frame used to design the trap [70]. The center of the trap door, at the bottom of the trap, is thus chosen to be $(0, 0, -1.5)$. Because the surface of the UCN$_\tau$ trap follows these radii, the simplest description of magnetic fields in the experiment utilizes a curvilinear coordinate system, $(\hat{\eta}, \hat{\zeta}, \hat{\xi})$. In this system, $\hat{\zeta}$ points perpendicular to the trap’s surface, $\hat{\xi}$ points tangentially...
along the trap in the direction of the holding field, and $\hat{\eta}$ points perpendicularly to the other two unit vectors. These coordinates can be related to the Cartesian coordinate by the transformations:

\begin{align*}
\eta(x, y, z) &= r(x) \times \arctan \left( \frac{x}{\sqrt{y^2 + z^2} - R(x)} \right) \\
\zeta(x, y, z) &= r(x) - \sqrt{x^2 + \left( \sqrt{y^2 + z^2} - R(x) \right)^2} \\
\xi(x, y, z) &= (R(x) + r(x)) \times \arctan \left( \frac{y}{z} \right).
\end{align*} \tag{5.5}

Given the local surface coordinates in equation (5.5), descriptions of the magnetic fields can then proceed by approximating the trap as locally flat. The UCN$\tau$ trap utilizes a curved Halbach array of permanent magnets to trap UCN. A Halbach array is a specific configuration of magnets, with adjacent magnets rotated by 90°[71]. Such a rotation of magnets increases the magnitude of $\vec{B}$ on one side while minimizing the field on the other side. In the curved coordinate system, the $\vec{B}$ of a Halbach array can be written as a Fourier series expansion[70]:

\begin{equation}
\vec{B} = \frac{4B_{\text{rem}}}{\pi \sqrt{2}} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n - 3} \left( 1 - e^{-k_n d} \right) e^{-k_n \zeta} \left( \sin k_n \eta \hat{\eta} + \cos k_n \eta \hat{\xi} \right). \tag{5.6}
\end{equation}

In equation (5.6), $B_{\text{rem}}$ represents the remnant strength of the permanent magnets, $d$ represents the thickness of the magnets, and $k_n = \frac{2\pi}{4\pi - 3} L$ incorporates the period, $L$, of one complete rotation in the magnetization. In UCN$\tau$, these values have been measured as $B_{\text{rem}} = 1.35$ T, $d = 25.4$ mm, and $L = 51.114$ mm[72]. Note that equation (5.6) violates Laplace’s equation, as the curvilinear coordinates ($\hat{\eta}, \hat{\zeta}, \hat{\xi}$) change as a UCN traverses the field. Since the field falls off much faster than the curvature of the array, this approximation should not lead to significant deviation from the expected behavior. The series expansion can be computed up to as many terms in $n$ as computing limits allow. Simulations in this work use $n = 3$, as higher order terms do not lead to significant
variations in the overall phase space evolution[72].

To avoid any field zeros that would lead to a spin flip, a set of 10 holding field coils encircle vacuum chamber that contains the Halbach array. These approximate a curved solenoid along the array, providing an additional field tangential to the Halbach field:

\[
B_\xi = B_0 \frac{r + R}{\sqrt{y^2 + z^2}} \xi. \quad (5.7)
\]

The total magnetic field strength can then be found by combining equation (5.6) and equation (5.7). Combining this with the gravitational potential energy, the total potential energy, \(V\), of a UCN in the trap can then be written as:

\[
V = \mu_n \sqrt{B_\eta^2 (\zeta, \eta) + B_\xi^2 (\zeta, \eta) + B_\xi^2 (x, y, z)} + m_ngz = \mu_n |\vec{B}| + m_ngz. \quad (5.8)
\]

A force on a single UCN can then be found by taking the gradient of the energy. This can be done by taking a partial derivative of the curvilinear magnetic field,

\[
\frac{\partial |\vec{B}|}{\partial [x, y, z]} = \frac{1}{|\vec{B}|} \left( B_\xi \left( \frac{\partial B_\xi}{\partial \zeta} \frac{\partial \zeta}{\partial [x, y, z]} + \frac{\partial B_\xi}{\partial \eta} \frac{\partial \eta}{\partial [x, y, z]} \right) + B_\eta \left( \frac{\partial B_\eta}{\partial \zeta} \frac{\partial \zeta}{\partial [x, y, z]} + \frac{\partial B_\eta}{\partial \eta} \frac{\partial \eta}{\partial [x, y, z]} \right) + B_\xi \left( \frac{\partial B_\xi}{\partial \zeta} \frac{\partial \zeta}{\partial [x, y, z]} + \frac{\partial B_\xi}{\partial \eta} \frac{\partial \eta}{\partial [x, y, z]} \right) \right). \quad (5.9)
\]

With the total force on a given UCN calculated, the motion of neutrons can then be calculated at any point in the trap. This magnetic field approximates the field in the trap, and a more accurate description of UCN trajectories could use a measured field model instead of the analytic model. This has not yet been incorporated into the simulations. Such a measured field map would require a method of producing continuous derivatives in order to calculate the forces on the UCN. The advantage of the analytic model is speed and scalability, as the magnetic field is smooth up to a peak trap height of 0.5 m.
Above 0.5 m, the curvilinear coordinate $\eta$ becomes imaginary, and as such it limits the maximum UCN energy available to these simulations to be $\sim 50$ neV.

5.2.3 Symplectic Integrator and Energy Conservation

Trajectories of UCN within the trap follow Hamiltonian mechanics. The Hamiltonian for a particle moving in a field, with position coordinates $q_i$ and momentum coordinates $p_i$, can be simply written as:

$$\mathcal{H} = V(\vec{q}) + T(\vec{p}).$$  \hspace{1cm} (5.10)

The Hamiltonian involves the potential energy, $V(\vec{q})$ described in equation (5.8), and the kinetic energy, $T(\vec{p}) = \frac{p^2}{2m_n}$. For a UCN in a known magneto-gravitational field, the equation of motion can then be written down with Hamilton’s equations:

$$\frac{dp_i}{dt} = -\frac{d\mathcal{H}}{dq_j} \quad \frac{dq_i}{dt} = \frac{d\mathcal{H}}{dp_i}. \hspace{1cm} (5.11)$$

For a MC simulation, the trajectory can be tracked by numerically stepping through equation (5.11). Simulated neutrons move around the UCN $\tau$ trap, with the goal of studying the most likely scenarios of loss and the details of phase space changes between short and long holds. A typical Runge-Kutta integration algorithm does not typically preserve all Poincare invariants, which could lead to a variation in the phase-space geometry as time elapses[73]. In particular, it does not conserve the energy of the neutron, which is critical for the working of the magnetic field in our trap. As we are interested specifically in changes in phase-space, UCN $\tau$ simulations need a different method of solving the equations of motion. A symplectic integrator utilizes an explicit fourth order algorithm, which preserves the phase-space density[74].

A symplectic integration algorithm can be seen in algorithm 6. The symplectic integration constants, $a_i$ and $b_i$, have many solutions which preserve phase space density.
Algorithm 6 Symplectic Integration Algorithm

1: initial position $t_0, \vec{x}_0, \vec{p}_0$
2: $H = V(\vec{x}_i) + T(\vec{p}_i)$
3: for $i = 1, 2, 3, 4$ do
4: \hspace{1em} $\vec{p}_i = \vec{p}_{i-1} - b_i \delta t \times \nabla_{\vec{x}} V(\vec{x})$
5: \hspace{1em} $\vec{x}_i = \vec{x}_{i-1} + a_i \delta t \times \nabla_{\vec{p}} T(\vec{p})$
6: \hspace{1em} $t_i = t_{i-1} + a_i \delta t$
7: end for

One set of constants, $a_i$, describes the motion through position space, while the other, $b_i$, describes momentum space changes.

<table>
<thead>
<tr>
<th>Symplectic Constant</th>
<th>Value (arb.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>.5153528374311229364</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-.085782019412973646</td>
</tr>
<tr>
<td>$a_3$</td>
<td>.4415830236164665242</td>
</tr>
<tr>
<td>$a_4$</td>
<td>.1288461583653841854</td>
</tr>
<tr>
<td>$b_1$</td>
<td>.1344961992774310892</td>
</tr>
<tr>
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<td>-.224819803079420806</td>
</tr>
<tr>
<td>$b_3$</td>
<td>.7563200005156682911</td>
</tr>
<tr>
<td>$b_4$</td>
<td>.3340036032863214255</td>
</tr>
</tbody>
</table>

Table 5.1: Symplectic integration constants, numerically optimized for energy conservation[75].

The numerical precision using the symplectic integrator is quantified by non-conservation of energy. Parameters of the integration model have been previously tested by studies of chaos in the trap. The performance of the symplectic algorithm depends on the input timestep for the simulation. For these simulations, a step size of 0.5 ms has been shown to have local energy fluctuations of $O(10^{-8})$ and a global energy drift of $O(10^{-11})$[72].

5.2.4 Neutron Generation

Trajectory simulations for the UCN$\tau$ trap are not concerned with the upstream motion of neutrons. Only a small fraction of UCN actually propagate from the source to successfully fill the trap, and thus even in simulations with millions of neutrons generated, trajectory simulations from the source would not reach the desired precision. As a result,
neutrons used for these simulations are generated in-situ, following reasonable probability distributions inside the trap. The choice of these models can then be tuned by fitting to data to reconstruct the initial conditions.

The analytic field model described in section 5.2.2 does not include the perturbations due to the TD. Normally during filling, the TD sits open to allow UCN to enter the trap. This creates a $15 \text{ cm} \times 15 \text{ cm}$ hole at the bottom of the trap, which removes a section from the Halbach array field in equation (5.6). To simulate the loading period, UCN are randomly generated using the xoroshiro1024* random number generator on a $15 \text{ cm} \times 15 \text{ cm}$ $(x, y)$ plane, at the minimum potential energy height, which to double precision is $z = -1.464413669130002 \text{ m}$[76].

Neutrons coming from a superthermal source follow a density distribution $\rho \propto E^{[13]}$. Prior upstream conditions, such as interactions with the guides or magnets, can selectively filter this energy distribution further. The expected energy distribution of trap-pable UCN in the trap thus follows a power distribution, $\rho \propto E^x$, where $x$ is a tuneable parameter corresponding to these upstream energy-dependent losses. Furthermore, neutrons leaving the guide with too low an energy would not be able to enter the real experiment, as they would be unable to enter through the trap door. To account for this, a low energy threshold cut, $E_{\text{min}}$, was also incorporated. Utilizing the Heaviside step function, this can be written as $\rho \propto \Theta (E - E_{\text{min}})$.

The TD region should have some combination of specular and diffuse scattering, leading to UCN entering with a specific angular distribution. In the case that the trap door is predominately diffuse, the angular emission spectrum of UCN entering the trap should follow a Lambertian distribution, $\rho \propto \sin (\theta) \cos (\theta)$. Here $\theta$ has been defined as the angle with respect to the $\hat{z}$ axis. As the trajectory simulations do not incorporate the effect of the trap door on filling, UCN entering the trap have initial trajectories somewhere between the diffuse case and a purely forward-directed, specular limit. To accommodate this, the angular density is weighted to give a more forward directed
Combining the previously defined terms, the overall phase space density generated in the simulations can be written as:

$$\rho (E, \theta) \propto \Theta (E - E_{\text{min}}) E^x \sin (\theta) \cos^{1+y} (\theta).$$  \hfill (5.12)

Simulations for the entire source and guide geometry have been investigated using the simulation package PENTrack, but these have not yet been compared to the initial conditions described here[77]. Nevertheless, the phase space distribution in equation (5.12) behaves similarly to the distribution of UCN inside the trap[72].

5.2.5 Detector Parameters

The interaction between UCN and various detectors uses a quantum mechanical multilayer model[13]. Each layer of the detector can be treated as a 1 dimensional quantum mechanical step function. In this limit, the reflection coefficient, $R$, for a given boundary can be written as:

$$R = \frac{-M_{21}}{M_{22}}. \hfill (5.13)$$

In equation (5.13), the matrix coefficients $M_{ij}^N$ for the $N^{th}$ layer has been multiplied across all layers, i.e. $M = M^N \times \cdots \times M^2 \times M^1$. These matrix terms can be explicitly written as the coefficients of the wavefunction and its derivative. By matching boundary conditions with the wavevector, $k_n' = \hat{z} \sqrt{\frac{2m}{\hbar^2}} (E_{\perp} - U_n)$, utilizing the complex potential $U_n = V + iW$, and defining a scaling factor $\gamma_n = k_{n-1}/k_n$, the transmission and reflection matrix can be written as:

$$M'' = \frac{1}{2} \begin{bmatrix} (1 + \gamma_n) e^{i(k_{n-1} - k_n)z_n} & (1 - \gamma_n) e^{-i(k_{n-1} - k_n)z_n} \\ (1 - \gamma_n) e^{i(k_{n-1} - k_n)z_n} & (1 + \gamma_n) e^{-i(k_{n-1} - k_n)z_n} \end{bmatrix}. \hfill (5.14)$$
The absorption probability, $\mu(E_\perp)$ for a UCN with perpendicular energy $E_\perp$ hitting such a multilayer surface can then be written as:

$$\mu(E_\perp) = 1 - |R| \quad (5.15)$$

The surface model for both the dagger and the active cleaner is a thin layer of $^{10}$B overlaying a 10 $\mu$m layer of ZnS:Ag. The $^{10}$B thickness for the dagger, $B_{th}$, can be tuned along with spectral parameters in order to more accurately model the detector parameters. UCN that are not absorbed by a detector are assumed to be scattered diffusely, as the surface roughness of the detector is unknown.

The dagger detector in the simulation is an infinitesimally thin sheet that activates whenever the UCN passes through the $y = 0$ plane. The lower edge of the detector fits the lower curve of the trap between $-0.3524 \text{ m} < x < 0.0476 \text{ m}$, while the upper edge has a flat top 0.2 m above the bottom of the dagger’s lowest point. In the event that a UCN passes through the dagger but does not get absorbed, the UCN gets reflected diffusely. Additionally, the dagger housing sits above the detector. The housing consists of two parts: a trapezoid, with height 14.478 cm, lower width 40 cm, and upper width 69.215 cm; and a rectangle with height 12.192 cm and width 69.215 cm. The dagger does not instantaneously move, instead progressing with an acceleration and deceleration of 6 m/s$^2$ to a maximum velocity of 1.6 m/s.

The dagger does not have uniform acceptance across the entire region. Along the bottom edge of the dagger, the light-collection is less efficient due to damage caused by dropping the dagger into the array. This can be modeled by introducing a reduced efficiency region with width $\zeta_0$. Within this region, the probability of neutron absorption varies linearly such that $\mu(\zeta, E_\perp) = \mu(E_\perp) \times \frac{\zeta}{\zeta_0}$ if $\zeta < \zeta_0$. Outside of this reduced efficiency region, the probability of absorption only depends on $E_\perp$.

The two cleaners in the experiment are modeled in a similar way to the dagger. An interaction occurs any time a UCN passes through a plane of constant $z$. During the
filling and cleaning times, the cleaner sits at a height of 38 cm above the bottom of the trap. For most of the holding period, the cleaners sit at 49 cm above the bottom of the trap. Analogous to the dagger, the cleaners move vertically with a constant speed of 5 m/s rather than instantaneously jumping between heights. The giant cleaner is treated as a solid sheet occupying the entire $y > 0$ half of the plane. The small cleaner is a 66.04 cm $\times$ 35.56 cm sheet centered at $(x, y) = (-5.95945, -62.19415)$ cm. Although in the actual trap the cleaners move slightly inwards with actuation, the simulated cleaners move purely vertically. The giant cleaner surface has a fixed efficiency; in the event that this efficiency is not 100%, the reflection is diffuse. The active cleaner surface, like the dagger, utilizes a $^{10}$B and ZnS:Ag multilayer quantum mechanical calculation. The $^{10}$B layer thickness for the active cleaner can be independently tuned or coupled to the dagger layer thickness.

The detector parameters are tuned so as to minimize the $\chi^2$ value between the measured data and the simulated unload on a 9-dip distribution. A simulated histogram fit against the actual data can be seen in figure 5.1. One of the leading uncertainties in generating the distribution comes from a lack of knowledge about the dagger movement profile. The dagger movement profile varies depending on many different local variables such as temperature and the lubrication of the motor. As a result, the simulated response varies most significantly from the actual dagger counting during times which the dagger moves. The most accurate set of coincidence parameters were tuned using the CMA-Evolution Strategy on a set of 2016 data, which has the same spectral parameters as 2017[72]. The dagger parameters correspond to a $^{10}$B thickness of $B_{th} = 5.6$ nm with a reduced efficiency region of 5.6 nm. The spectrum has an energy weighting of $x = 1.2$ and an angular weighting of $y = 0.28$. Given the fixed spectrum parameters, the simulations can then be used as a tool to investigate sources of UCN loss.
Figure 5.1: Comparison between 20 s holding times for a background-subtracted 9-dip unload and the trajectory simulations in 2018. The timing profile of dagger movement varies on a run-by-run basis. For these two histograms, $\chi^2/NDF \approx 4$. The regions with the largest residuals can be seen as times at which the dagger moves. Variations in the tunable parameters provide changes to the draining time at each step, as well as shifts in the overall energy distribution.

5.3 PSEUDODATA SIMULATIONS

5.3.1 Overview

An additional set of MC methods must be used to characterize detector efficiencies. In order to measure the neutron lifetime to a high precision, the ideal analysis requires
forming coincidences between PMTs. However, any coincidence algorithm will have a variable counting efficiency, which could couple to rates in the detector and thus bias the lifetime. The structure of reconstructed neutron events has been previously discussed in section 4.4. In particular, the telescoping coincidence algorithm used for this analysis can be found in section 4.4.4. The algorithm described has variable “coincidence parameters:” the “initial window,” \( W_I \); the “telescoping window,” \( W_T \); the “prompt window,” \( W_P \); and the “photon threshold,” \( \gamma_T \). These must be appropriately chosen so that the event reconstruction can be well understood.

The extended length of coincidence events can lead to both deadtime and long-tail pileup effects. The addition of deadtime will lower the efficiency of counting at high rates, as two or more quickly arriving UCN could be counted in a single “event.” A longer deadtime then serves to increase the measured lifetime for a short-long pair. Long tails of the UCN event can increase the counting efficiency, since whichever form the coincidence algorithm takes could potentially trigger on both the initial coincidence and on later photons. This increased efficiency at high rates will decrease the apparent lifetime. Real events, with a combination of deadtime and pileup, will thus require corrections in both directions. Rather than calculating a lifetime and subsequently applying a Rate Dependent Effects (RDE) shift, the RDE can be quantified for each coincidence. The mechanisms for evaluating these have been previously discussed in section 4.4.6 and section 4.4.7.

Studies of the effectiveness of these algorithms’ rate dependent effects, and the response of various coincidence algorithms on the lifetime, requires pseudodata MC simulations. Such a simulation generates a known number of events, formed by resampling event probability distributions from real data. The model used for constructing events will be discussed in section 5.3.2. This MC simulation quantifies the deadtime and pileup effects due to pulse structure and optimizes coincidence parameters to characterize the lifetime shift from rate-dependent effects. In section 5.3.3, the pseudodata simulations
can be used to validate the effectiveness of our coincidence weightings for establishing RDE corrections.

5.3.2 Event Model

Simulated UCN events must be re-sampled from the actual data. Within a region with consistent dagger conditions, probability distribution histograms can be generated by summing many events together. These histograms are taken from isolated coincidence events, with a minimum of 50 $\mu$s distance between coincidence events. A slightly looser coincidence model, with $W_T = 2000$ ns and $\gamma_T = 4$, is utilized to generate these histograms. The PMT waveforms can be analytically modeled as a hyperexponential function, the sum of multiple exponentials:

$$P(t) = \sum_{i=1}^{N} \frac{\alpha_i e^{-t/\tau_i}}{\tau_i}. \tag{5.16}$$

Such a probability distribution is normalized so long as:

$$\int_{0}^{\infty} P(t) dt = 1. \tag{5.17}$$

The deviation from such a hyperexponential is greatest at the beginning of the coincidence event, as the response of the PMT is distorted by deadtime and potential afterpulsing effects. A piecewise function can thus be formed to extrapolate the initial distribution of events out to $t \to \infty$. Early photons generated in coincidences can incorporate such short-term fluctuations, while a hyperexponential with $N = 3$ can be used to expand the timing of summed events out to $t \to \infty$.

The SIMD-Oriented Fast Mersenne Twister (SFMT) random number generator provides individual, random, photon counts for each coincidence event[78]. A Poisson number of counts, $N_S$ and $N_L = \text{Int}(N_S e^{-1530/\tau_n})$, are generated for each “run.” The starting time for each coincidence is resampled from summed short holding time un-
load histograms. Two random numbers are used to generate the first photon of each coincidence in order to randomize the time within each histogram bin. An additional random number determines which PMT starts the first photon, with a such a weighting coming from data. The number of photons generated in each PMT due to a coincidence event can be resampled from the pulse height distribution, which has been previously shown in figure 4.19. Given a known number of photons to be generated, the timing information can be reconstructed from the histograms previously shown in figure 4.14. These can then be separated out based on the PMT upon which the original photon is counted.

On top of the coincidence events generated in the previous process, two types of background events are generated. Following the same resampling procedure as the foreground events, coincidence background events can be injected on top of the resampled pseudodata. The starting time for each coincidence background event comes from a homogeneous Poisson distribution. Single photon background events with a fixed rate are also injected into the simulated distribution. After producing foreground and the two types of background events, the generated counts in each PMT are filtered to remove events that would be impossible due to the 16 ns hardware deadtime.

5.3.3 Validation of Coincidence Algorithm Corrections

A neutron lifetime measurement will be affected by two competing rate-dependent effects, deadtime and long-tail pileup. The choice of coincidence algorithm parameters will affect the relative contributions of these two effects. A longer window will increase the amount of correction required to account for deadtime, as an individual coincidence lasts for longer. This increases the length of events and thus increases the amount of time in which two UCN cannot be seen in short succession. A shorter window will increase the amount of correction required to account for pileup, as more of the long tail of the ZnS:Ag glow will not be part of the coincidence. This will potentially cause retriggering
on coincidence events, if they fail to absorb enough of the coincidence event. Pileup can be exacerbated if the photon threshold is too low, as this will increase the number of possible retriggers.

![Simulated Lifetime Shift](image)

Figure 5.2: Rate-dependent shift in the coincidence lifetime. Each point on the 2D histogram has a fixed uncertainty of ±0.05 s. As the $W_T$ increases, the deadtime becomes longer, increasing the calculated lifetime. Increasing the $\gamma_T$ reduces the potential for retriggering, which decreases the calculated lifetime.

In order to investigate the competing effects, the resampled event data described in the previous section can generate many events to see the shift on the lifetime. The resampled data can be generated with the same rates of foreground and background data as across the multiple dagger conditions. The resulting lifetime shifts due to varying the coincidence parameters can be seen in figure 5.2. As expected, increasing the photon threshold decreases the measured lifetime, as the effect of pileup is diminished. And similarly, increasing the telescoping window increases the effects of deadtime when compared to pileup. There is always a single photon deadtime correction of about
\[ \Delta(\tau_{\text{RDE}}) \sim 0.15\text{s}, \] as individual photons have a hardware deadtime of 16 ns. For the coincidence parameters chosen in the analysis, the lifetime shift is \(-1.14 \pm 0.05\) s. Using the coincidence weighting algorithm described in section 4.4 corrects for a lifetime shift of \(-1.16 \pm 0.03\) s, in agreement with the simulation prediction. Based on this, the coincidence reconstruction for our chosen coincidence algorithm can be validated from simulations to the level of 0.05 s.
6.1 INTRODUCTION

A bottle-type experiment does not directly measure \( \tau_n \). The lifetime reconstructed in chapter 4 is not the lifetime of the free neutron, \( \tau_n \), but is instead the lifetime of neutrons in the UCN\( \tau \) trap, \( \tau_{\text{meas}} \). In the event of external loss mechanisms besides \( \beta \)-decay, the neutron lifetime can be reconstructed by subtracting various interaction lifetimes in the trap:

\[
\tau_n = \left( \frac{1}{\tau_{\text{meas}}} - \frac{1}{\tau_{\text{depol}}} - \frac{1}{\tau_{\text{pk1}}} - \frac{1}{\tau_{\text{gas}}} - \ldots \right)^{-1}.
\] (6.1)

A neutron counted in a short holding time that would be otherwise lost during a similar long holding time would bias the lifetime. The reported neutron lifetime, then, must incorporate either corrections or systematic uncertainties due to non \( \beta \)-decay loss mechanisms. A combination of MC trajectory simulations, previously introduced in section 5.2, and data-driven methods can be used to constrain variations between short and long holding times. Some variations in the lifetime due to analysis choices have been discussed in previous chapters. Shifts in the measured lifetime due to background reconstruction, described in section 4.5, and normalization models, found in section 4.6, will not be further enumerated here.

This chapter will instead focus on explicit sources of neutron loss. Uncertainties due to depolarization have not changed from previous UCN\( \tau \) results. The data sets taken in 2017 and 2018 contain no new systematic investigations of this loss mechanism, and thus section 6.2 will review previous constraints. UCN interactions with ambient gasses, in section 6.3, will introduce a loss rate than can be calculated for each run. This loss rate
is shared between analyzers, although the methods by which an overall lifetime shift can be calculated might differ. Rate dependent effects, already partially investigated through MC methods in section 5.3, will be quantified in section 6.4. The addition of the small active cleaner has allowed for additional constraints on the effects of overthreshold neutrons, which will be discussed in section 6.5. Phase space evolution between short and long holding times, while not explicitly an additional source of loss, can lead to a shift in the counting time between short and long holding times. Limits on these effects can be found in section 6.6. Finally, the effects of a contaminant aluminum block in the experiment will be described in section 6.7. Many systematic constraints are statistically limited, and thus future iterations of UCN\(\tau\) will gain systematic precision as more data is taken.

### 6.2 DEPOLARIZATION

Prior to entering the trap, UCN pass through the magnetic fields described in section 3.2.2, in order to polarize incident neutrons into a purely low-field seeking state. Neutrons in the high-field seeking state would not levitate in the magnetic field but would instead be pulled towards the trap surface. While the initial polarization of UCN entering the trap is not 100\%, any high field seeking UCN would be lost either during filling or cleaning, and thus would not contribute to a source of loss.

Quantum mechanical effects could, however, contribute to the potential depolarization of previously trapped neutrons. In the presence of an external magnetic field \(B\), the two spin eigenstates have a potential energy difference given by \(\Delta E = 2\mu_n |B|\). This energy difference goes to zero in the absence of any field, and thus a “field zero” present in UCN\(\tau\) would cause spontaneous depolarization. External holding field coils provide a constant field with maximum magnitude \(|B_{\text{max}}| = 6.4\) mT to maintain polarization. Due to the asymmetric, nontrivial magnetic fields in the UCN\(\tau\) Halbach array and the experimental area, certain regions of the trap could have an unforeseen cancellation of
magnetic fields. As UCN fill the available volume of the trap, such a field zero could slowly be sampled on long timescales. Additionally, UCN inside the trap are moving through varying magnetic fields, with small deviations from adiabatic behavior. Because the spin state of UCN contains both low and high field seeking components, movement through a non-uniform magnetic field can also cause spontaneous depolarization[79]. Simulations by Steyerl et al. have shown that the magnetic configuration of the UCN$\tau$ field leads to a measured trap lifetime, $\tau_{\text{meas}}$, differing from the actual lifetime, $\tau_n$, with a depolarization perturbation approximately following a power law[80]:

$$\frac{1}{\tau_{\text{meas}}} \approx \frac{1}{\tau_n} + \frac{B_{\text{max}}^2}{B^2\tau_{\text{depol}}}. \quad (6.2)$$

The depolarization lifetime, $\tau_{\text{depol}}$, can then be measured by varying the nominal holding field strength $B$ away from the maximum holding field strength $B_{\text{max}}$ and fitting to $\tau_{\text{depol}}$. A previous UCN$\tau$ data set, taken in 2015, varied the holding field coil strength from 0.5 mT to the maximum possible field of 6.8 mT. Fitting the lifetimes from these variable holding field configurations to the power law gave a loss lifetime of $\tau_{\text{depol}} = 1.1^{+4.4}_{-0.5} \times 10^7 \text{ s}[56]$. The relationship in equation (6.2) holds for magnetic fields $\geq 5 \text{ mT}$, and below this field the depolarization simulated by Steyerl falls below the power law[80]. However, as the magnetic field decreases, the probability of field zeros appearing inside the trap increases and dominates potential depolarization effects.

The 2017 and 2018 data sets did not dedicate any additional runs to investigate uncertainties due to depolarization. Instead, UCN$\tau$ only ran the experiment at the maximum available holding field strength. As a result, the systematic uncertainty due to depolarization in this work will keep the previous value of $+0.07 \text{ s}$. As UCN can only be lost due to depolarization, this effect can only decrease the lifetime. Based on this, the systematic uncertainty applies in just one direction.
6.3 GAS SCATTERING CORRECTION

Trace amounts of residual gas in the vacuum chamber can lead to upscattering or absorption of UCN. Interactions between UCN and room temperature gas almost always impart more energy than the UCN\(\tau\) trapping potential of \(\sim 50\) neV. The rates at which UCN interact with residual gasses in the imperfect vacuum of the trap must therefore be well-characterized.

The UCN\(\tau\) chamber has two cryopumps attached to the east endcap of the vacuum vessel. For most of the 2017 and 2018 run cycles, the trap was capable of a minimum pressure between \(1 \times 10^{-7}\) Torr and \(5 \times 10^{-7}\) Torr. The trap door movement causes a small spike in pressure at the beginning of the hold. Trap door actuation causes the surrounding o-rings to gradually deteriorate, which increases the effects of these pressure spike. Additionally, any changes made to the trap required opening the trap to atmosphere. Following this, the UCN\(\tau\) trap took more than 24 hours to reach its minimum pressure. As a result, some data was taken during elevated vacuum conditions. On the north and the south sides of the experiment, two Cold Cathode (CC) gauges register the pressure in the trap. These pressures can be read into the Environmental Monitoring System (EMS), allowing pressure measurements of the trap on a continuous basis.

Previous UCN\(\tau\) publications have investigated the cross section of UCN and various contaminant gasses that could be found in the vessel[81]. Water and heavy hydrocarbons have very high UCN cross sections, and can significantly scatter trapped neutrons[82]. During 2017 and 2018, multiple Residual Gas Analyzer (RGA) samples determined the relative composition of contaminant gasses in the vacuum. Based on these scans, the predominant source of UCN gas interactions comes from water in the trap, particularly after long periods with the trap open. Unlike the trap pressure measurements with the CCs, the RGA scans were not continuous. The light produced by the RGA increased the background rate of the two dagger PMTs. In between each RGA measurement, the
relative gas composition is assumed to vary linearly.

Each gas species has a known density, $N_{\text{gas}}$ and average velocity, $\bar{v}_{\text{gas}}$. The total cross-section for a given gas, $\sigma_{\text{tot}}$, is a combination of the absorption and upscattering cross sections, $\sigma_{\text{abs}}$ and $\sigma_{\text{up}}$. An individual UCN-gas molecule interaction has a cross-section inversely proportional to the incident velocity of the UCN. The time constant of interaction, $\tau_{\text{gas}}$, can then be determined as:

$$\frac{1}{\tau_{\text{gas}}} = \sigma_{\text{tot}} N_{\text{gas}} \bar{v}_{\text{gas}}. \quad (6.3)$$

For each run, the pressures are recorded by the CC and the relative fractions of water, $N_2$, $O_2$, and heavy hydrocarbons are determined from the RGA data. Then, the $k^{\text{th}}$ run will have a tabulated mean loss rate $L_k = \tau_{\text{gas},k}^{-1}$. During the 2017 and 2018 dataset, these mean loss rates were shared between each analyzer. The total loss rate in the trap will be a combination of $\beta$-decay and gas scattering, with an expected number of counts during the unload $N_{\tau,k} \sim e^{-t \times (\tau_n^{-1} + L_k)}$. The neutron lifetime, $\tau_n$, has already been incorporated into the normalization algorithm. To reconstruct neutrons lost by gas scattering, the expected number of counts can be multiplied by the tabulated loss rate, $e^{-t \times (L)}$.

Applying this additional source of loss for each run provides a total shift due to residual gas interactions. Each run has an uncertainty in the loss rate due to pressure variations and uncertainty of the gas species that is heavily correlated with other runs. To account for these correlations, the upper and lower bound of the overall lifetime shift can be determined by incorporating the upper and lower bounds of $L_k$. In 2017 this additional source of loss leads to a lifetime shift of $0.132^{+0.059}_{-0.038}$ s, while in 2018 the shift is $0.057^{+0.021}_{-0.022}$ s. Combining these two corrections across the entire dataset provides an overall gas scattering correction of $0.11^{+0.05}_{-0.03}$ s. Gas scattering is the primary non-$\beta$-decay source of loss in the UCN$\tau$ trap, and so this work will report an overall lifetime shifted by this correction.
The uncertainty due to RDEs can be found by studying the effect of coincidence parameters on the lifetime. The two competing effects of deadtime and pileup will change the measured number of coincidences in both short and long holding times, as has been previously seen in section 5.3. In this analysis, the initial window has been chosen as $W_I = 50$ ns, the photon threshold is $\gamma_T = 8$, and the telescoping window is $W_T = 1000$ ns. The prompt window has been fixed such that $W_P = W_T$. Variations from these parameters could potentially change the measured lifetime. The coincidence reweighting algorithm can be turned on or off for any combination of these coincidence parameters. The lifetime can be calculated while varying any of these to determine an uncertainty due to RDEs.

The results of such a parameter sweep can be seen in figure 6.1. The BR3 supercomputer ran 63 complete instances of the paired lifetime analysis described in chapter 4. Each of these used different coincidence parameters to calculate all relevant dagger counts, including the unload and background events. A common runlist, with the same normalization monitor counts, was used for each lifetime. Certain coincidence parameters failed to evaluate the RDE correction; short telescoping windows ($W_T < 600$ ns with high photon thresholds ($\gamma_T > 12$) were unable to parse all the “good” data. Lifetime calculations utilizing these coincidence parameters, $W_T < 600$ ns and $\gamma_T > 12$ were thus excluded from the analysis. A high threshold counting for a short period of time could potentially lead to divide-by-zero errors in calculation of backgrounds or pileup effects. As the parameter sweep approaches these limits, the effectiveness of the pileup correction comes into question. Outside of this region, the correction behaves as expected, with a broad region where the measured, RDE corrected, lifetimes are comparable.

The uncertainty due to RDE effects can be found by histogramming the corrected coincidence parameter scan. Such a histogram can be seen in figure 6.2. Each lifetime
Figure 6.1: Paired lifetime calculated from the entire dataset with varying coincidence parameters, using a linear interpolation scheme between 100 ns steps in $W_T$ and single photon steps in $\gamma_T$. The upper plot shows the general effect of RDEs, as no correction to the coincidence algorithm has been made. The lower plot shows the same parameter sweep but with the coincidence weighting algorithm applied. While the upper plot ranges $\pm 2$ s across the entire dataset, the lower plot only varies by $\pm 0.3$ s, indicating the effectiveness of the coincidence weighting. Both plots have been scaled so that 0 s corresponds to the corrected value for $W_T = 1000$ ns and $\gamma_T = 8$ ns.
Figure 6.2: Histogram of the paired lifetime calculated with a 2-dimensional scan of varying coincidence parameters, with $600 \text{ ns} \leq W_T \leq 1400 \text{ ns}$ and $6 \leq \gamma_T \leq 12$. A total of 63 unique combinations of parameters were considered. The measured lifetimes can be reasonably well-described by a Gaussian distribution, using the mean and standard deviation of the calculated lifetimes.

measured, after correcting for RDE effects, behaves similarly to other results. This distribution can be described, with a $\chi^2/\text{NDF} = 0.834$, by a Gaussian with mean and standard deviation of the RDE corrected lifetimes. The standard deviation, $\sigma_{RDE} = 0.13 \text{ s}$, of this distribution provides the uncertainty due to coincidence parameters and rate dependent effects. Unlike many of the other systematic effects, this uncertainty can act in either direction.
6.5 OVERTHRESHOLD NEUTRONS

6.5.1 Overview

The UCN\textsubscript{\texttau} trap cannot hold every neutron that exits the source and propagates through the guides. The guides between the source and roundhouse are coated with NiP, which has an $E_f = 213$ neV. However, the surface of the magnets themselves, as well as much of the vacuum vessel, has an Aluminum coating, which has a Fermi potential of only $E_f = 54$ neV. The height of the trap itself is only 50 cm, so a UCN with a high enough energy could reach the edge of the trap and potentially spill out of the magnetic chamber. Additionally, the magnetic field at the surface of trap, $\sim 1$ T, can only levitate UCN with an energy about 60 neV, above which energy neutrons can interact with the magnet walls. High energy UCN can thus potentially escape the trap and contribute to a lifetime bias.

The spectrum of UCN during ordinary production must be cleaned such that neutrons untrappable by the above means do not enter the lifetime calculation. During the cleaning period, the two cleaners lower down to 38 cm, the same height as the first dip in the dagger movement pattern. During the holding time, the two cleaners sit at 43 cm above the bottom of the trap. The giant cleaner features a sheet of polyethylene, which efficiently upscatters UCN. The smaller active cleaner has a similar detector coating to the dagger detector. Both of these remove a high fraction of incident neutrons.

In the event of slow changes in the phasespace density of neutrons, UCN with an energy high enough to reach the cleaners might not actually be removed prior to the hold. Such “insufficient cleaning” would manifest in an excess in high energy UCN during short holding times. These UCN would gradually shift into trajectories which would allow them to be cleaned. Additionally, even in the case that every UCN begins with an energy below the trapping potential, the energy distribution of trapped UCN might not be constant. Neutrons without enough energy to reach the cleaners might
instead gain energy through some “heating” mechanism. In this case, newly boosted overthreshold UCN would appear in the long holding times.

The three-dip distribution for production running allows quantitative measurements of these high energy UCN. The dagger begins the counting period by moving to the cleaning height of 38 cm for 40 s. During the 2018 run cycle, the AC was lowered into the trap during dip 1 in addition to the dagger. The AC counts overthreshold UCN more efficiently than the dagger and thus provides more stringent limits on this population of UCN. Investigations of overthreshold UCN utilized special running conditions, where the cleaners remained raised during the entire run. In this way, the shift in lifetimes due to overthreshold UCN can be directly investigated.

6.5.2 Overthreshold Simulations

Trajectory simulations can give information about the relative efficiency of counting neutrons in peak 1 between the AC and the dagger. A set of simulations, each with 2048000 trajectories, investigated the expected response of the two detectors to uncleaned UCN. The simulation utilized the best fit for 2017, with an upper energy cutoff of 50 cm. Higher energy UCN can be expected to appear in the actual trap, but the analytic form of the magnetic fields prohibits simulating these.

The simulated response of the two detectors can be seen in figure 6.3. The AC, when lowered during counting, can be seen to be significantly more sensitive to the high energy population. For a 20 s holding time, the AC sees 98291 UCN compared to just 1884 UCN in the dagger. This changes slightly for a 1550 s holding time, where the AC detects 103993 UCN instead of the dagger’s 1041 UCN. Based on the simulations, then, 98.1% of nominally uncleaned UCN should be seen in the AC compared to the dagger. Similarly, 99.0% of potentially heated UCN will be seen in the AC instead of the dagger.

The trajectory MC simulations cannot track trajectories with very high energies. As a result, the overthreshold UCN systematic purely based on simulations undershoots
Figure 6.3: Histogram of UCN hits from trajectory MC simulations without cleaning. Both the AC and dagger detectors lower to 38 cm from the bottom of the trap at $t = 0$. Signals have been recorded in both the dagger and AC detectors, for short and long holds. Numbers in the legend indicate the total number of UCN counted on each detector. Very minor differences between 20 s and 1550 s holding times can be seen. The motion of the dagger in peaks 2 and 3 moves some UCN into trajectories where they hit the AC.

the expected shift in lifetime. The simulation predicts a lifetime shift after not cleaning UCN of 5.22 s. This is shorter than the actual measured lifetime shift in the uncleaned data, of $O(20)$ s. This suggests that many neutrons actually lost during the hold begin with an energy higher than the trajectory simulations can track, $51 \text{ neV} < E < 168 \text{ neV}$. Alternately, the simulation has deficiencies with modeling phase-space evolution, which will be described in section 6.6.2. These could preferentially move UCN into trajectories where they no longer interact with the dagger, the detector used for lifetime calculation. Either of these possibilities would nevertheless lead to a signal in the AC, and thus study
of this detector’s counts provide an important limit on overthreshold counts.

6.5.3 Dagger Overthreshold Correction

During normal running, the dagger counts at 38 cm above the bottom of the trap for the first 40 s of the unload. UCN at this height should have already been cleaned out prior to the hold, and thus any significant number of UCN counted in peak 1 can be a cause for concern. As described above, this dagger counting can be inefficient. Nevertheless, for data taken in 2017 the AC was not lowered during the counting period. As a result, the only tool for investigating peak 1 counting in 2017 comes from the dagger.

Two limits can be taken to benchmark the knowledge of overthreshold UCN. The dagger unload curve at 38 cm follows an exponential decay. During the rest of the unload, some number of overthreshold UCN will continue to be counted. Extrapolating out the UCN counted during peak 1 with the unload time constant can provide a minimum on the overthreshold neutron counts. The opposite limiting case assumes that the dagger counting time constants for overthreshold UCN are the same as other UCN during the unload. In this case, the fractional components of the UCN at each counting height should be constant for all dagger conditions.

An investigation of both of these parameters can occur by investigating the counting properties of the dagger without any cleaning. Some assumptions must be made about the counting conditions of dagger events. The uncleaned overthreshold UCN must have the same counting profile as the cleaned overthreshold UCN. In both 2017 and 2018, dedicated no-cleaning runs allowed the investigation of these counting fractions.

The rate seen in a detector is some combination of $\eta$ and $\tau_n$. In particular, the actual number of counts seen in the detector, $N_{\text{obs}}$, will be scaled by the counting rate, $R_{\text{count}}$ and the rates of $\beta$-decay, $R_\beta$:

$$N_{\text{obs}} = N_{\text{tot}} \frac{R_{\text{count}}}{R_{\text{count}} - R_\beta}. \quad (6.4)$$
This equation can be rearranged to account for just the observable values in the experiment. The unload of the dagger can provide a measured draining time for each dip, \( \eta_j \). The inverse of these draining times, follows \( R_{\text{count}} = R_\eta - R_\beta \). This allows equation (6.4) to be rewritten in terms of things we can observe:

\[
N_{\text{tot}} = N_{\text{obs}} \frac{\eta^{-1}}{\eta^{-1} - \tau_n}.
\]  

(6.5)

Each peak counts for a finite amount of time, with some decay that fits an exponential:

\[
N_{j,\text{obs}} = \int_{t_{j,0}}^{t_{j,f}} N_j e^{-t'/\eta_j} dt' = \eta_j N_j \left[ e^{-t_{j,0}/\eta_j} - e^{-t_{j,f}/\eta_j} \right].
\]  

(6.6)

Solving the previous equations for each dip, and utilizing only observable values from the unload, the total number of overthreshold UCN, \( N_{\text{tot,over}} \), can be determined using:

\[
N_{\text{tot,over}} = \frac{N_{1,\text{obs}}}{\eta_1 \left( 1 - e^{-t_{1,f}/\eta_1} \right)} \left[ \frac{\eta_1^{-1}}{\eta_1^{-1} - \tau_n^{-1}} \right] \sum_j \eta_j \left( e^{-t_{j,0}/\eta_j} - e^{-t_{j,f}/\eta_j} \right).
\]  

(6.7)

The draining time has two components. Part of the draining time comes from a geometric height dependence of neutrons able to reach the dagger. The other component of the draining time comes from a phasespace and energy dependent component, which would vary for different populations of neutrons with energy \( E \). In the case that within a given dip the phasespace distribution is constant, these components of the draining time can be separated out to become:

\[
\eta_j = \epsilon \left( h_j \right) \eta_{\text{det}} \left( E \right).
\]  

(6.8)

The previous UCN\( \tau \) publication used a peak 1 correction assuming a single constant
exponential. This provides a minimal limiting case, equivalent to using a constant value for \( \epsilon (h_j) \eta_{\text{det}} (E) \). An alternate limit would be to assume a constant \( \eta_{\text{det}} (E) \), and assume variations in the draining time come purely from height dependence. This would overestimate the number of overthreshold UCN, but could be calculated from known values in dagger counting.

![3-Step Neutron Counting, 2017](image)

Figure 6.4: Uncleaned dagger unload from the beginning of 2017 compared to the cleaned dagger unloads. Each peak in the unload has been fit to a single exponential draining time based on the uncleaned data. It takes a longer amount of time to count higher peaks. This is because the dagger has a lower acceptance of counts at lower positions.

Uncleaned data was taken in both 2017 and 2018. The summed, background-subtracted uncleaned data can be used to find the relevant time constants for the dagger draining time conditions. For one subset of the data, the summed unloads and time constants can be seen in figure 6.4. With these time constants, the number of extra UCN during the unload can be calculated by using equation (6.7). The average number of UCN in each
run can then be added back into the unload, taking the upper and lower limits from the two limiting cases described above. This procedure can be repeated for both short and long holding times, adding the counts back to either the short or long unloads. Adding these counts back into the lifetime calculation can then provide a shift in the lifetime.

UCN in peak 1 after short unloads correspond to “insufficient cleaning.” In 2017, the overthreshold UCN present in the short unloads results in a lifetime shift of \( \Delta(\tau_{\text{meas}}) = 0.02 \pm 0.01 \) s. In 2018, the short unloads provide a lifetime shift of \( \Delta(\tau_{\text{meas}}) = 0.02 \pm 0.02 \) s. A similar procedure, but done using the long holding times, provides a limit due to “UCN heating” during storage. These respective lifetime shifts are \( 0.07 \pm 0.01 \) s in 2017 and \( 0.03 \pm 0.01 \) s in 2018. The overall correction due to these effects will be chosen as the maximum of these values for each case. Any of these counts provides a limit on the respective loss mechanism; UCN cannot be added back into the trap. This means that the relevant corrections must be unidirectional. Using the dagger, the systematic uncertainty due to insufficient cleaning will thus be reported as \(+0.04\) s. The related correction for heated UCN will be \(+0.08\) s. Due to the poor counting efficiency near the top of the trap, these uncertainties might not adequately describe the sources of loss.

6.5.4 Active Cleaner

The AC provides an additional constraint on the number of overthreshold UCN. A complementary procedure to section 6.5.3, but using counts on the AC, can be applied to determine a systematic uncertainty. In 2018, the AC was lowered into the trap during the first peak, at the same time as the dagger. As previously described in section 6.5.2, the AC has a significantly higher efficiency of counting neutrons in this region. Additionally, it consistently samples the same region. While the dagger has some unknown, and changing, acceptance of high-threshold UCN, the AC keeps the same counting profile. Thus there is no need to apply multiple time constants as with the dagger. This means that the AC provides a more robust estimate of overthreshold neutron counting.
Figure 6.5: Background-subtracted active cleaner counts during “peak 1.” The AC counts have been generated requiring a $\gamma_T = 4$. Short holding times have a slight excess of counts in peak 1, suggesting some small amount of uncleaned UCN. The reported mean and uncertainty in the legend come from the mean and standard deviation of the distribution; the actual uncertainty must incorporate additional uncertainty due to the backgrounds.

UCN absorbing on the AC can be isolated by utilizing the integrated window algorithm described in section 4.4.9. Integrated window counts in the AC were generated using a slightly looser parameter than the dagger counting. While the telescoping window remained at 1000 ns, the photon threshold for an event was lowered to 4 PEs.

The AC backgrounds, which have a rate $\sim 1.5$ Hz, are not very well known. A significant contribution to the AC background comes from cross-talk with the dagger. The AC PMTs can see the glow of the dagger’s ZnS:Ag scintillator, but this signal depends on the relative positions of the two detectors. Dedicated background runs do not utilize the same motion patterns as production runs. To accommodate this, background estimates
came from both the holding time and the end of the run. These two configurations provide minimum and maximum bounds on the potential backgrounds. During the holding time, both the dagger and cleaner are retracted, and so the interaction between the two detectors should be minimal. At the end of counting, both the dagger and cleaner are fully lowered and thus have the highest amount of cross-talk.

The distribution of background-subtracted AC counts can be seen histogrammed in figure 6.5. The mean number of neutrons seen in the active cleaner in peak 1 can be used, in conjunction with the uncleaned data, to determine the excess of counts across the entire unload. Since the acceptance of the AC does not change, the single exponential limit of equation (6.7) is appropriate. These additional counts can be added to the unload of each run and propagated through the lifetime calculation. The upper and lower limits of the background can be used to determine a region of best fit for the systematic uncertainties. After the short holding times, the active cleaner sees an average of \(0.72 \pm 1.46\) UCN. This corresponds to a lifetime shift due to uncleaned neutrons of \(0.04 \pm 0.07\) s. For long holding times, the active cleaner instead sees an average of \(-0.08 \pm 1.50\) UCN. The associated shift in lifetime due to heated UCN is \(-0.02 \pm 0.09\) s. The upper limits of these lifetimes can be used as an uncertainty value of the lifetime; there cannot be a net gain in UCN due to heating or insufficient cleaning. Similarly, there is not enough information to provide a non-zero correction to the lifetime, and thus only an uncertainty will be reported. Using the AC, then, this analysis reports an uncertainty of \(0 + 0.11\) s for insufficient cleaning and \(0 + 0.07\) s for heated UCN.

The most conservative choice of systematic uncertainty can be chosen from a combination of the AC and dagger peak 1 corrections. The reported heated UCN uncertainty, \(+0.08\) s, comes from the dagger, while the uncleaned UCN uncertainty, \(+0.11\) s, comes from the AC. By improving the background estimate on the AC or by combining the AC and dagger counts, future analyses might be able to improve upon these systematic uncertainties.
6.6 PHASE SPACE EVOLUTION

6.6.1 Overview

A bottle-type measurement requires identical measurement conditions between short and long holding times. However, the detection efficiency for UCN in the dagger detector varies depending on the perpendicular energy, as $P_{\text{abs}} \sim E_\perp$. The distribution of $E_\perp$ crossing the central plane is one part of the overall phase space distribution of neutrons in the trap. In the case that the phase space density of trapped UCN is not constant, the mean $\langle E_\perp \rangle$ or the variance $\sigma(E_\perp)$ might change. If either of these distribution parameters change between a short and long holding time, the detector efficiency is no longer constant.

A “Phase Space Evolution (PSE)” effect could lead to a bias in the measured lifetime, $\tau_{\text{meas}}$.\(^4\) If detection is more efficient after short holding times than after long holding times, $\tau_{\text{meas}} < \tau_n$. The opposite case, where counting after long holding times is more efficient, leads to $\tau_{\text{meas}} > \tau_n$. A brief overview of the predicted PSE behavior from trajectory simulations will be found in section 6.6.2. Quantitative evidence of PSE can be seen in relative fractional changes from each dagger peak. Studies of these can be seen in section 6.6.3. The actual uncertainty due to PSE can be isolated by calculation of the lifetime using variable arrival times for each run. This method will be discussed in section 6.6.4. These various methods can then provide systematic uncertainty limits of the variable efficiency of counting in UCNτ.

6.6.2 Phase Space Evolution Simulations

The trajectory simulations described in section 5.2 give some benchmarks for PSE. A short-long pair with holding times 20 s and 1550 s, each with 2048000 MC trajectories,

---

\(^4\)When referring to systematic uncertainties in the UCNτ experiment, “Phase Space Evolution” will be used for the change in counting efficiency between short and long holding times. Overthreshold UCN also provide an uncertainty due to changes in the phasespace distribution, but this uncertainty will be attributed to their source of loss.
was utilized to investigate systematic shifts in the dagger efficiency. These used a maximum generated UCN energy of 40 neV in order to reduce the effects of insufficient cleaning and maximize statistics. With this cut, of the initial 2048000 trajectories, 55452 or 2.7% were still removed prior to the holding period. Using the thicker $^{10}$B coating in 2017, there is a difference of only 3 UCN between short and long holding times. This corresponds to a fractional difference of $1.5 \times 10^{-6}$, and a simulated PSE effect of 0.003 s.

A more realistic limit on the uncertainty due to PSE comes from calculating the simulated Mean Arrival Time (MAT), $\langle t \rangle$. For the two simulations described above, the unweighted mean of all detected events can be calculated to determine $\langle t \rangle$. For the 20 s holding time, $\langle t \rangle = 71.086 \pm 0.007$ s. This slightly differs from the 1550 s holding time, which has a $\langle t \rangle = 71.221 \pm 0.007$ s. The two MATs from the simulations suggest a slight lifetime shift due to PSE. Using these two numbers in a lifetime calculation gives a $\langle t \rangle$ lifetime shift of $\Delta(\tau_n) = 0.08 \pm 0.008$ s.

Accuracy of the trajectory simulations of PSE in the UCN$\tau$ trap is limited by the ideal field expansion. The existence of small deviations in the magnetic field can be shown to increase chaotic motion in the trap. A direct measurement of “phasespace regeneration” can be seen by counting UCN at a lower position than the normal cleaning height. Immediately after cleaning, the dagger sits at the normal peak 2 height, 25 cm above the bottom of the trap, for 150 s. After this, the dagger is raised for a variable holding time before being returned to 25 cm. This removes a region of phasespace; any neutrons counted after returning to 25 cm come from phasespace evolution. Figure 6.6 shows a comparison between simulated data and real PSE regeneration data. Two data-taking periods were used: the 2016 run cycle was taken prior to this work; the 2018 run cycle had the RH and thus a significantly different filling profile. The MC simulation does not adequately describe the PSE regeneration. The addition of chaotic elements, such as the aluminum block described in section 6.7 or additional microphonic heating, increases PSE regeneration. As a result, the UCN$\tau$ trap most likely has additional sources of PSE
Figure 6.6: Simulated phase space regeneration counts compared to actual data taken from the previous run cycle, 2016, and the current run cycle, 2018. UCN were removed by the dagger at 25 cm from the bottom of the trap, and then counted some time later. The raw simulation data does not adequately describe the phase space evolution of the trap. The addition of microphonic heating or the aluminum block provides additional PSE, which helps explain the actual data taken.

Unload Distributions

UCN have a total energy comparable to the gain in gravitational potential energy across the entire trap. Because of conservation of energy, UCN at a higher point in the trap will have less kinetic energy. Since the detection probability depends on the transverse energy of incident UCN on the dagger, counting at a higher position reduces the dagger’s efficiency. During the counting period, the dagger lowers in multiple discrete steps. Each step probes a different population of UCN in the trap. The fraction of UCN counted at
various heights can then be used to reconstruct the dagger efficiency.

Figure 6.7: Percentage of counts in unload dip 2. The increased efficiency of the dagger in 2018 leads to an increased percentage of counts in dip 2. The presence of the aluminum block causes the discrete steps in the middle of 2017. In 2018, the fraction of counts in dip 2 of the distribution is consistent between short and long holding times, signaling a minimal amount of phase space evolution. In 2017, the fraction of counts changes between short and long holding times, primarily due to the presence of the aluminum block.

A shift in counting efficiencies manifests as a change between fractions in the unload counts between short and long holding times. On a run-by-run basis, the percentage of counts in the unload appears as in figure 6.7. There are roughly 3 major regions visible in the data. The shift at run 9600 comes from the change in dagger boron thickness between years. A thicker dagger detection layer means more neutrons absorb on the first bounce, as neutrons need less perpendicular energy to be captured. Another section occurs between runs 4711 and 7326; this is due to the presence of the contaminating aluminum block. Qualitatively, for most running conditions the percentage of counts in each dip
remains the same between short and long holding times.

Figure 6.8: Summed unload counts from 9-dip distributions in 2018, background subtracted and summed to 1 s bins. All holding lengths used for production are consistent with no shift. The residual plot shows the maximum fractional deviation between short and long holding times is $\sim 0.002 \text{ s}^{-1}$ in a 1 s bin. Across the entire unload, the integrated fractional residual is $f_0 = 1.73 \times 10^{-14}$, which provides a negligible shift in the lifetime. Taking instead the integral of the absolute value of the residual, the fractional shift is $f_a = 2.78 \times 10^{-4}$.

A quantitative estimate of the effect of PSE can be found by summing over the unload distribution in each dagger condition. Production runs normally utilize a 3-dip distribution for neutron counting. In 2018, a small subset of the runs utilized a 9-dip unload pattern instead, which provides a different sampling of the phasespace. A comparison between various holding times in such an unload can be seen in figure 6.8. The fractional shift between 20 s and 1550 s holding times has a maximal residual of $f_a = 2.78 \times 10^{-4}$. If the change in counting efficiency is exactly equal to the maximal residual quoted here, the lifetime shift can be estimated by multiplying the counts during a long holding time
by $1 \pm f_a$. Using this shift in efficiency, the 9-dip unload data suggests a shift due to PSE of $0 \pm 0.14$ s. This result can be taken as a worst-case scenario.

Figure 6.9: Summed unload counts from 3-dip distributions with the same dagger configuration as figure 6.8, background subtracted and summed to 1 s bins. All holding lengths used for production are consistent with no shift. The residual plot shows that the maximum fractional deviation between short and long holding times is only $\sim 0.001$ s$^{-1}$ in a 1 s bin. Across the entire unload, the integrated fractional residual is $f_0 = 6.19 \times 10^{-15}$, about a factor of 2 better than the shift seen in the 9-dip distribution. Taking instead the integral of the absolute value of the residual, the fractional shift is $f_a = 1.09 \times 10^{-4}$.

The statistical reach of the 9-dip unload is not as good as the normal 3-dip unload pattern. The fractional shift of a 3-dip unload can be seen instead in figure 6.9. Since significantly more data was taken with 3-dip data, the fractional shift between short and long holding times will be reduced. As in the 9-dip case, modifying the measured long counts by the maximal fractional shift of $f_a = 1.09 \times 10^{-4}$ accounts for the worst case shift in counting efficiency. The additional statistical precision of the 3-dip data...
provides a limit on the shift due to phase-space evolution of $0 \pm 0.06 \text{ s}$. Again, the assumptions made for calculation of a lifetime shift, that the rate is directly dependent on the maximum uncertainty, will significantly overestimate the uncertainty due to PSE. Summing the total distribution does not adequately describe discrete changes in the detector. A change in the height-dependent background model, for example, could slightly shift the fraction of counts appearing in each lifetime band.

### 6.6.4 Mean Arrival Times

An alternate, more precise, method of calculating the uncertainty in lifetime due to PSE comes from calculating the shift in MAT, $\langle t \rangle$. The fraction of counts seen in a dip after a holding time $t$ can be arbitrarily written as $f(t, \epsilon)$. This fraction depends on the neutron energies, $\epsilon$, and should be roughly the same between short and long holding times. The difference between two holding times can be described by incorporating an additional perturbation:

$$f(t_L, \epsilon) = f(t_S, \epsilon) + \delta f(t_L, \epsilon). \quad (6.9)$$

Given an arbitrary energy-dependent draining time for a given dip, $\eta(\epsilon)$, a Taylor expansion around the mean arrival time MAT for a lifetime pair leads to a first-order correction of:

$$\frac{\langle t \rangle_L - \langle t \rangle_S}{\ln \frac{Y_S}{Y_L}} = \tau_n \left[ 1 + \frac{1}{t_L - t_S} \left( \frac{\int \delta f(\epsilon) \eta(\epsilon) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) \right]. \quad (6.10)$$

It can be shown, as in appendix D, that this first-order correction cancels the shift in lifetime due to variable counting efficiency. Each run has statistical fluctuations in the measured UCN arrival times, which provides additional uncertainty due to the MAT. The statistical spread in MAT at long holding times becomes the dominant source of uncertainty over any biases added by higher-order terms excluded from equation (6.10), which would be of $\mathcal{O}(10^{-3}) \text{ s}$.
Figure 6.10: Mean arrival time of neutrons during unload, subtracted from the holding time for each run. A more efficient detector, as in 2018, corresponds to an earlier mean arrival time. The mean arrival time is $\sim 70$ s after the dagger starts moving, which corresponds to the time the dagger reaches the bottom of the trap. The difference between short and long holding times is minimal, but the long holding times have a greater spread in their values.

Calculating the lifetime using the MAT for each run, when compared to calculating the lifetime from nominal holding times, thus provides a shift due to PSE. Data from 2017 provides a shift of $0.018 \pm 0.002$ s, while 2018 provides a shift of $0.010 \pm 0.003$ s. Combining these two years provides an overall lifetime shift of $0.016 \pm 0.002$ s. The uncertainty due to this can then be used as the uncertainty due to MAT, significantly below the uncertainty provided by other means.
A significant amount of the data taken in 2017 contains an accidental material contamination. On December 1st of the 2017 run cycle, an aluminum spacer block was found to have fallen from the small cleaner frame, which can be seen in figure 6.11. This block had been unnoticed for weeks, as the trap had not recently been opened for routine maintenance. The block falling into the trap occurred early enough in the run cycle that any shift in lifetime was within statistical uncertainty. In order to maintain vacuum,
the trap itself is rarely opened during production running, and so visual inspection of
the trap was rarely done.

The presence of the contaminant led to two potential problems in this set of run data. As
the block came from the small cleaner, the change in the cleaning position could
lead to an additional uncertainty in the efficiency of cleaning. However, the previous
determination of the overthreshold UCN systematic does not depend upon the actual
cleaning efficiency. As a result, there is no evidence of an additional insufficient cleaning
systematic during periods with the Al block. More importantly, the block itself will
both reflect and upscatter UCN. Reflections off the block’s Aluminum surface changes
the phase space distribution. Most importantly, the interaction of UCN with the block
leads to a potential source of loss. To check the effect of this, the UCN\(\tau\) Monte Carlo
simulation, discussed in section 5.2, incorporated the block to study the resultant change
in lifetime.

6.7.2 Black Block

After the aluminum block was located, the effect of the block on the data was investi-
gated by coating the block with polyethylene. As polyethylene upscatters UCN with a
very high efficiency, this served to magnify any sources of loss. This “black block” was
carefully put into the same position as the original contaminant block. By measuring the
resulting shorter neutron lifetime, the rate of neutrons hitting the block could be deter-
mined, allowing an estimate of bounds on the lifetime shift from the aluminum block.
9-dip and 3-dip runs were taken with the polyethylene coated block, utilizing varying
holding times. The dominant source of loss in the trap with a polyethylene block is no
longer \(\beta\)-decay. As a result, the “long runs” for data with the black block were reduced to
600 s, rather than the more typical 1550 s during normal production. The results of these
runs can be seen in figure 6.12. Runs with longer holding times remove a higher number
of UCN from the higher energy sections of the unload. This provided a deficiency in the
first two dips for the dagger unload data.

Figure 6.12: 9-step data taken with the polyethylene coated aluminum block in place. The rate has been normalized to have an integral of 1000. The lower plot shows the difference between 20 and 600 s holding times. Longer holding times show a reduction in the number of counts at high energies, corresponding to the first two peaks of the unload, indicating a preferential removal of high-energy UCN by the presence of the block. This also demonstrates the magnetic field’s shielding of the block from low-energy UCN. Later peaks include lower energy UCN and thus do not show depletion in longer holding times, and so have a relatively higher rate when normalized.

6.7.3 Block in Simulation

The effect of the Aluminum block on the lifetime can be quantified by using the MC simulations presented in section 5.2. The block was modeled in the MC as a detector with a known position and rotation. At each timestep, the simulation checked to see if the neutron trajectory passed through a 3-dimensional surface bounding the block. The position and rotation were calculated in Mathematica, using the position of the block and
magnets in figure 6.11 as a reference. Each magnet has a known size, and the position of the aluminum block can thus be determined by counting the magnets on top of which the block had landed. Additionally, the block must be slightly raised; the four corners of the block rest on the bottom of the trap. In the simulation, the block is thus modeled as a $2.54 \times 2.54 \times 1.27$ cm$^3$ rectangle centered at the coordinates:

\[
\begin{align*}
    x_{\text{mid}} &= 0.217588 \text{ m} \\
    y_{\text{mid}} &= 0.126445 \text{ m} \\
    z_{\text{mid}} &= -1.436798 \text{ m}.
\end{align*}
\] (6.11)

The 3 dimensional rotation of the block can be described by a $3 \times 3$ rotation matrix, $\overline{R}$. This rotation matrix is:

\[
\overline{R} = \begin{pmatrix}
    0.498185 & -0.746320 & -0.441382 \\
    0.807037 & 0.585238 & -0.078665 \\
    0.317023 & -0.317022 & 0.893864
\end{pmatrix}. \quad (6.12)
\]

The block can have one of two surface models. The first model mimics the polyethylene-coated block by removing any UCN that interact with the simulated block. The second, more realistic case, treats the aluminum block quantum mechanically with an energy-dependent probability of loss per bounce, $\mu (E_{\perp})$. The Fermi potential of aluminum has a real component $\text{Re}(E_f) = V = 54$ neV and an imaginary component, denoted by $f = \text{Im}(E_f)/\text{Re}(E_f) = 2.25 \times 10^{-5}$[13]. The probability of loss per bounce can then be calculated:

\[
\mu (E_{\perp}) = 2f \left( \frac{E_{\perp}}{V - E_{\perp}} \right)^{1/2}. \quad (6.13)
\]

The timing distribution of UCN hits on the block was determined by simulating $10^6$ trajectories with the block as an additional detector. Any UCN trajectory that passed through the block counted as a “hit,” saving the timestamp, the position, and the mo-
mentum. After this hit, the UCN undergoes diffuse reflection off the block’s surface. A single trajectory can hit the block multiple times, with subsequent block hits recorded as well. After the simulation, each recorded hit is assigned a random number between 0 and 1. If the number generated is below the calculated UCN loss probability from equation (6.13), that UCN can then be removed.

Figure 6.13: Distribution of the first time a simulated UCN trajectory hits the aluminum block during a normal run. The peak at 150 s corresponds to the end of filling the trap; UCN in the simulation are not tracked during the entire filling time, but instead generated with a variable time profile. Many UCN are removed during filling or close to the beginning of the holding time, before a 20 s holding time.

Because the block is only a half inch high, the magnetic field in the vicinity of the surface shields the block from low energy neutrons. As such, a minimum energy of about 15 neV is required to actually hit and scatter off the block. This means the block preferentially scatters only high-energy neutrons. Higher energy neutrons can penetrate further into the field, and so a small fraction of neutrons can reach the side of the block
instead of the face. The timing and energy distribution of UCN hitting the block can be seen in figure 6.13 as predicted by the MC simulation. Fitting the time of the initial hit, integrated over the model energy distribution, to an exponential gives a time constant of the black block of $\tau_{\text{block}} = 275.3 \pm 27.7$ s, with $\chi^2/\text{NDF} = 0.96$. This interaction time constant can then provide some constraints on the shift of the neutron lifetime.

### 6.7.4 Lifetime Shift

A simple model for investigating the lifetime shift assumes a constant probability of loss-per-bounce with the interaction time constant, $\tau_{\text{block}} = 275.3 \pm 27.7$ s. For aluminum, the loss-per-bounce is $\mathcal{O}(10^{-4})$. The perpendicular energy of UCN incident on the Al block should be between $0 \leq E \leq 35$ neV, since the maximum energy will be reduced by magnetic shielding. However, the angular distribution of UCN trajectories hitting the block cannot be easily modeled, and thus the MC predicted $\tau_{\text{block}}$ differs from the measured $\tau_{\text{block}}$ of $\sim 400$ s. There is also uncertainty on the effective probability of loss per bounce.

Solving equation (6.13) while accounting for the distribution in $E_\perp$ requires some assumptions. Rough bounds on the UCN loss per bounce, $\mu$, can be found by looking at some mono-energetic cases. If all incident UCN have energies where $E_\perp = 35$ neV, the loss per bounce is $\mu = 0.6 \times 10^{-4}$. In the case that only half of the kinetic energy is directed perpendicular to the block's surface, $E_\perp = 17.5$ neV and $\mu = 0.3 \times 10^{-4}$. Additional surface roughness could contribute to more loss than the measured loss-per-bounce. A conservative estimate could be found by taking symmetric uncertainties about these monoenergetic cases, or $\mu = 0.9 \times 10^{-4}$. These cases provide a range of $\mu$ between $0.3 \times 10^{-4} \leq \mu \leq 0.9 \times 10^{-4}$.

Using this range of losses, the loss time due to the Al block can be found to be $3.059 \times 10^6 < \tau_{\text{Al}} < 9.177 \times 10^6$ s. This loss rate can be combined with a lifetime of $\tau_{\text{MC}} = 880$ s, to find the measured lifetime value $\tau_{\text{meas}}^{-1} = \tau_{\text{MC}}^{-1} + \tau_{\text{Al}}^{-1}$. This suggests a
shift in the lifetime between 0.08 s and 0.25 s. This shift should be taken as a worst-case scenario, as it has minimal information about the energies used to determine the loss-per-bounce. Furthermore, it assumes the trajectories able to see the Aluminum block are constant with time. This estimation should be combined with a more detailed accounting of the loss per bounce of trajectories in UCN$\tau$.

It should be noted that the interaction time constant depends on the initial spectrum of the block. Varying the initial spectrum of the trajectory MC, and using the actual loss function of UCN on Aluminum provides a more realistic method for determining the shift in lifetime due to the aluminum block. The results of the MC using $10^6$ trajectories were reweighted to incorporate uncertainty in the spectrum. The three spectral fit parameters, $E_{\text{min}}$, $x$, and $y$, can be varied using the energy spectrum equation (5.12). These parameters have been varied using 5 values for each parameter near the global minimum, leading to a total of 125 spectra scanned. Each reweighted, simulated 9-dip unload has been fit to 20 s black block data, with a $\chi^2$/NDF calculated. Spectra where the fit between the simulated and measured data has a $\chi^2$/NDF $> (\chi^2$/NDF)$_{\text{min}} + 1$ between the MC and the actual data have been rejected.

The lifetime shift due to this physical model of loss-per-bounce comes out to $0.08 \pm 0.04$ s. This number is shorter than the worst-case scenario presented above, as neutrons which are not lost can be scattered into regions where they no longer interact with the block. The simulated model of the trap does not fully account for the PSE in the trap. The shift in lifetime due to a real aluminum block will lie somewhere between the upper limit of the worst-case scenario and the lower limit of the simulations. Based on these potential variations due to spectral evolution, the correction due to the aluminum block will be reported as $0.14 \pm 0.10$ s.

The overall lifetime shift due to the Aluminum block can be calculated by adding 0.14 s to each paired lifetime when the block is in the trap. Calculating a paired lifetime, but now including this correction, will thus raise the total lifetime by 0.0646 s. The data
containing the Aluminum block has a paired statistical uncertainty of ±0.463 s. The total paired statistical uncertainty, including the block, is ±0.315 s, and the statistical uncertainty excluding the block is ±0.339 s. Using typical uncertainty propagation on the section containing the block, the uncertainty on the section with the block thus raises to 0.474 s. Incorporating this additional uncertainty, the inflated paired uncertainty for the entire run cycle has now been raised to 0.318 s. Isolating the effect of this increased uncertainty, 0.045 s, can then provide the systematic uncertainty due to the block. Based on this, the effect of the aluminum block on the combined data set gives a lifetime shift of +0.06 ± 0.05 s. This value can thus be added as a correction, in parallel with the other systematic uncertainties described here.

6.8 SUMMARY OF RESULTS

Previous sections have described the various sources of loss in the UCNτ experiment requiring a systematic correction or uncertainty. This analysis reports a systematic correction of +0.17±0.21 −0.16 s. An overview of these systematic corrections can be seen in the results table 6.1.

<table>
<thead>
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<th>Systematic</th>
<th>Correction (s)</th>
<th>Uncertainty (s)</th>
<th>Section</th>
</tr>
</thead>
<tbody>
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<td>+0.07</td>
<td>6.2</td>
</tr>
<tr>
<td>Gas Scattering</td>
<td>+0.11</td>
<td>+0.05, −0.03</td>
<td>6.3</td>
</tr>
<tr>
<td>Rate Dependent Effects</td>
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<td>±0.13</td>
<td>6.4</td>
</tr>
<tr>
<td>Heated UCN</td>
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<td>+0.08</td>
<td>6.5</td>
</tr>
<tr>
<td>Uncleaned UCN</td>
<td>0</td>
<td>+0.11</td>
<td>6.5</td>
</tr>
<tr>
<td>Aluminum Block</td>
<td>+0.06</td>
<td>±0.05</td>
<td>6.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>+0.17</td>
<td>+0.21, −0.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Systematic sources of uncertainty in the 2017-2018 UCNτ analysis. Corrections must be applied for gas scattering and the aluminum block, while the other systematic uncertainties assume no correction is required. The heated and uncleaned UCN corrections use the most conservative determination of the systematic effect. Using a combination of the AC and dagger could improve upon these. The aluminum block correction noted here is for the entire dataset. Phase space evolution is not listed in this table, as the effects can be corrected by using the MAT instead of the nominal holding times.

Applying this correction to the reported global uncertainty leads to an overall uncer-
tainty on this experiment of $\Delta(\tau_n) = \pm 0.26 \text{(stat.)}^{+0.21}_{-0.16} \text{(sys.)}\, s$. The correction required for the lifetime is comparable to the statistical uncertainty, and well below most other neutron lifetime experiments. The overthreshold UCN and phase space evolution corrections can be expected to decrease as the systematic reach of the experiment increases. Similarly, future iterations of the UCN$\tau$ experiment should not be expected to have the contaminant of the aluminum block.
7.1 SUMMARY OF RESULTS

After two years of taking data, the UCN\(\tau\) experiment has measured a blinded neutron lifetime of \(\tau_n = 887.82 \pm 0.26^{+0.21}_{-0.16} \) s. This unprecedented precision has been achieved through the use of a novel bottle formed by asymmetric permanent magnets. The magnetic nature of the trap reduces the need to correct for UCN losses to determine \(\tau_n\). The small loss rate of the trap, predominated by gas scattering, leads to a lifetime shift smaller than the statistical uncertainty. The lifetime reported in this work are compared to two other independent analyses of this set of UCN\(\tau\) data for consistency checks. These other analyses were developed independently, and utilize different methods of run selection, event generation, background subtraction, and normalization. Despite the variation in methods, these analyses agree with each other with a maximum discrepancy of only 0.1 s. Furthermore, these three analyses have been tested on subsets of the total 2017-2018 data set, providing confidence in their methods. The analyses are only partially correlated; each analyzer uses a unique run quality determination and a different event reconstruction algorithm.

After ensuring the three analyses agreed, the blinding factor was unencrypted and found to be \(f = 1.01144\). The analysis described here can thus be unblinded to \(\tau_n = 877.78 \pm 0.26^{+0.21}_{-0.16} \) s. An unweighted mean of the three independent analyses is used to determine the neutron lifetime from this run of UCN\(\tau\). The overall statistical uncertainty is also given by an unweighted mean; the systematic uncertainties come from the most conservative values across the three analyses. Combining these lifetimes in this manner gives an overall result of \(\tau_n = 877.75 \pm 0.27^{+0.22}_{-0.16} \). Compared to the previous UCN\(\tau\) result,
this is a factor of $2.5 \times$ improvement in statistical uncertainty and an improvement of $1.5 \times$ in systematic uncertainty[56].

The neutron lifetime determined with UCN$\tau$ has now reached a precision of $4 \times 10^{-4}$. This precision in measuring $\tau_n$ provides a competitive, independent, method of investigating the unitarity of the CKM matrix. Given the Particle Data Group (PDG) value of $\lambda = -1.2756 \pm 0.0013$ this UCN$\tau$ result finds a value of $V_{ud} = 0.9746(3)$. This value agrees with the prediction from unitarity, and raises slight tension with the value of $V_{ud}$ extracted from $0^+ \rightarrow 0^+$ decays. Improved measurements of angular decay parameters should be done in conjunction with additional lifetime data taken by the UCN$\tau$ experiment. A determination of $V_{ud}$ extracted purely from the neutron would help constrain future tests of physics beyond the SM.

7.2 FUTURE PROSPECTS

The results reported here make the UCN$\tau$ experiment the world’s most precise measurement of $\tau_n$. Nevertheless, UCN$\tau$ can still continue to improve to provide further constraints on $\beta$-decay theory. Future iterations of the UCN$\tau$ experiment should attempt to maximize statistics while minimizing systematic effects, in order to reach the stated precision goal of $\pm 0.1$ s, or $1 \times 10^{-4}$ relative precision.

One of the largest uncertainties of this work was the RDEs caused by the coincidence algorithm. Due to instabilities in the single photon backgrounds, the reported $\tau_n$ in this work utilized a coincidence method of counting. The next largest uncertainties are due to overthreshold UCN. Future running of the UCN$\tau$ experiment should attempt to mitigate these uncertainties. One of the leading sources of systematic uncertainty in this data set was caused by the contaminant Aluminum block. This mistake should be avoided in future iterations of the experiment.

An improvement to the dagger can reduce systematic effects due to RDEs. One possible studied improvement has been the introduction of a LutetiumYttrium Oxy-
orthosilicate (LYSO) dagger. Inefficiencies in counting UCN come from the long tail of the ZnS:Ag scintillator. An LYSO scintillator counts coincidence events significantly faster than ZnS:Ag, and thus would resolve individual neutron events faster. This would reduce the need for a large deadtime and pileup correction in coincidence counting. Such an LYSO dagger is under development; its implementation presently suffers due to increased sensitivity to position-dependent backgrounds. An alternate method to deal with higher rates would be to segment the dagger into many smaller sections. Each piece of a segmented dagger would have its rate reduced due to the smaller area associated with each detector. In order to reach the desired precision of future lifetime experiments, one of these methods of improving counting efficiency must be implemented.

Many of the systematic limitations of UCN$\tau$ can be mitigated by increased neutron statistics. If UCN$\tau$ measured $O(10) \times$ more UCN per unload, the improved signal-to-noise ratio would lead to less uncertainty. This could even allow the use of a single photon analysis, mitigating the large uncertainty due to RDEs. A net increase in counts should be proportional across the entire UCN energy spectrum. If the overthreshold UCN uncertainties are presently overestimated, the increased number of total counts would not necessarily be seen in peak 1. As a result, data-driven methods of investigating overthreshold UCN will be improved as the number of trapped UCN increases.

The source produces many more UCN than what are trapped and counted in a typical UCN$\tau$ unload. UCN densities upstream of the trap are orders of magnitude higher than the UCN densities in the trap. Many of these UCN are trappable in the guides but not by the magnetogravitational trap, and so cannot be used to improve statistical reach. However, a significant population of otherwise trappable UCN cannot enter the trap via the TD; the efficiency of neutron loading is much smaller than anticipated. To address this issue, a novel “elevator” loading mechanism, the improved “UCN$\tau+$” experiment, is under development. This would adiabatically move UCN from the guides into the trap, providing a lower loss rate than the fast movement of the TD. Such an
elevator would increase the amount of UCN available to be counted in the trap without redesigning the magnetic field.

Further in the future, the addition of higher magnetic fields with a reworked geometry would provide a higher trapping potential for UCN. This “UCNτ2” apparatus would make more efficient use of the UCN spectrum from the source, as more higher-energy neutrons are produced. To reach magnetic fields above 2 T, such an improved array would require superconducting magnets. In addition to re-designing the storage geometry, superconducting magnets would require novel mechanisms for loading the trap. The elevator loading studied for UCNτ+ could be utilized for this purpose.

The UCNτ apparatus has overcome several engineering challenges to measure the neutron lifetime. Lessons learned from the present set of data will provide areas of study for future works. In the next 5 years, UCNτ+ hopes to reach a 0.1 s total uncertainty, with ever-diminishing uncertainties beyond that. These improvements will continue to provide stringent tests of the Standard Model and search for new physics beyond the Standard Model at energy levels comparable to high-energy colliders.
REFERENCES


[46] J Byrne et al. “Determination of the electron-antineutrino angular correlation coefficient $a_0$ and the parameter $|\lambda| = |g_A/g_V|$ in free neutron $\beta$-decay from measurements of the integrated energy spectrum of recoil protons stored in an ion trap”. In: *Journal of Physics G: Nuclear and Particle Physics* 28.6 (Apr. 2002), pp. 1325–1349.


APPENDIX A
PRODUCTION RUN BREAKS

This table illustrates the various detector gain shifts that correspond to discrete changes in the trap. Each of these regions must be analyzed separately, as they have different normalization responses.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>4230</td>
<td>Beginning of 2017</td>
</tr>
<tr>
<td>4304</td>
<td>Dagger PMT Gain Shift</td>
</tr>
<tr>
<td>4391</td>
<td>Abnormal Source</td>
</tr>
<tr>
<td>4415</td>
<td>“Normal” Source</td>
</tr>
<tr>
<td>4711</td>
<td>Aluminum Block falls in</td>
</tr>
<tr>
<td>5453</td>
<td>Abnormal Source</td>
</tr>
<tr>
<td>5475</td>
<td>“Normal” Source</td>
</tr>
<tr>
<td>5713</td>
<td>Foil Monitor Gain Shift</td>
</tr>
<tr>
<td>5955</td>
<td>Dagger PMT Gain Shift</td>
</tr>
<tr>
<td>6126</td>
<td>Dagger PMT Gain Shift</td>
</tr>
<tr>
<td>6429</td>
<td>Dagger PMT Gain Shift</td>
</tr>
<tr>
<td>6754</td>
<td>Foil Monitor Gain Shift</td>
</tr>
<tr>
<td>6930</td>
<td>Foil Monitor Gain Shift</td>
</tr>
<tr>
<td>7326</td>
<td>Aluminum Block Removed</td>
</tr>
<tr>
<td>7490</td>
<td>Dagger PMT cooling</td>
</tr>
<tr>
<td>7612</td>
<td>Dagger PMT cooling</td>
</tr>
<tr>
<td>9767</td>
<td>Beginning 2018</td>
</tr>
<tr>
<td>9960</td>
<td>RHAC Installed</td>
</tr>
<tr>
<td>10936</td>
<td>RH Detector Installed</td>
</tr>
<tr>
<td>10988</td>
<td>RHAC Raised</td>
</tr>
<tr>
<td>11085</td>
<td>RHAC Lowered</td>
</tr>
<tr>
<td>11669</td>
<td>RH Re-installed</td>
</tr>
<tr>
<td>12516</td>
<td>RHAC Lowered</td>
</tr>
<tr>
<td>13209</td>
<td>Dagger PMT1 shift</td>
</tr>
<tr>
<td>13307</td>
<td>Dagger PMT shift</td>
</tr>
</tbody>
</table>

Table A.1: Discrete changes in the trap requiring a RB

In addition to the detector gain shifts noted in table A.1, the source changed drastically between certain Melting and Refreezing (MRF) cycles. Most of these happened in
the beginning of 2017, and thus were probably due to unfamiliarity with the source. Certain source conditions nevertheless caused drastic changes in UCN production. These early melting and refreezing cycles can be found in table A.2. Table A.1 includes two of these “abnormal sources,” each of which lasted for \( \sim 20 \) runs.

<table>
<thead>
<tr>
<th>First Run of Source</th>
<th>Included in RB?</th>
</tr>
</thead>
<tbody>
<tr>
<td>4391</td>
<td>Yes</td>
</tr>
<tr>
<td>4415</td>
<td>Yes</td>
</tr>
<tr>
<td>5050</td>
<td>No</td>
</tr>
<tr>
<td>5220</td>
<td>No</td>
</tr>
<tr>
<td>5334</td>
<td>No</td>
</tr>
<tr>
<td>5453</td>
<td>Yes</td>
</tr>
<tr>
<td>5475</td>
<td>Yes</td>
</tr>
<tr>
<td>5584</td>
<td>No</td>
</tr>
<tr>
<td>5617</td>
<td>No</td>
</tr>
<tr>
<td>5635</td>
<td>No</td>
</tr>
</tbody>
</table>

Table A.2: Major MRFs at the beginning of 2017

Across the entire run cycle there are \( \sim 100 \) individual source MRFs. As many of these sources only contain one or two 20 s holding time runs, using every possible MRF as a RB is impractical. Only the most discrepant sources were included in the list of RBs.
APPENDIX B
DAYTIME AND NIGHTTIME BACKGROUNDS

Background rates in the trap do not agree between the daytime and nighttime running. This leads to difficulties in using a single photon analysis. Previously, in equation (4.32) the shift in lifetime due to background subtraction has been formulated in the format of signal-to-noise ratio on the counts. This is numerically solved in figure B.1.

Figure B.1: Solution of equation (4.32) for both additional shift in the backgrounds (blue) and additional uncertainty on lifetimes (orange). The green lines indicate a 0.1 s uncertainty in the lifetime.

The following tables record the averaged rates at various detector heights in the trap. In 2017, we expect to count 20000 UCN after a 20 s holding time. The photon amplitude should be $\sim 50$ photons /UCN. The total photon signal will then be $\sim 10^6$ counts, with
60% of those counts appearing in PMT1. Based on the rates measured in table B.1, the beginning of 2017 sees a rate difference of 2.223 Hz in PMT1 and 0.650 Hz in PMT2. Putting these values into equation (4.32), the difference between daytime and nighttime rates contributes an additional uncertainty of 0.98 s and 0.42 s for PMT1 and PMT2 respectively. This compares to a smaller shift for coincidence counting of 0.06 s, which is comparable to the statistical background uncertainty that we attribute to coincidence counting. Taking the worst case scenario, the latter section of 2017, the difference in measured lifetime between daytime and nighttime becomes 3.32 s for PMT1 and 4.57 s for PMT2. For coincidence counting, the heavily suppressed signal to background ratio provides a shift of merely 0.13 s. For a single PMT counting mode, the extra fluctuations in backgrounds here must be appropriately de-weighted.

The backgrounds measured in UCN\(\tau\) can be estimated from the end of the run, so fluctuations at the end of the run can be relatively easily accounted for. Where the background model runs into difficulties is when calculating height dependencies. The peak 2 and peak 1 background rates, as well as the time dependent background rates at the top of the trap, have a much greater level of uncertainty in their background rate. These uncertainties propagate through the height dependent background calcula-

Table B.1: Rates in background sections at bottom of the trap

<table>
<thead>
<tr>
<th>(k_i)</th>
<th>(k_f)</th>
<th>Type</th>
<th>PMT 1 (Hz.)</th>
<th>PMT 2 (Hz.)</th>
<th>Coinc. (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4200</td>
<td>7612</td>
<td>Day</td>
<td>144.079 ± 0.654</td>
<td>93.519 ± 0.663</td>
<td>0.137 ± 0.002</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Night</td>
<td>146.927 ± 1.149</td>
<td>95.391 ± 1.245</td>
<td>0.158 ± 0.007</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Prod.</td>
<td>146.302 ± 0.848</td>
<td>94.169 ± 0.933</td>
<td>0.142 ± 0.003</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Day</td>
<td>130.342 ± 0.548</td>
<td>93.667 ± 0.505</td>
<td>0.144 ± 0.002</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Night</td>
<td>140.100 ± 1.058</td>
<td>103.518 ± 1.087</td>
<td>0.161 ± 0.017</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Prod.</td>
<td>137.923 ± 0.740</td>
<td>100.641 ± 0.738</td>
<td>0.154 ± 0.004</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Day</td>
<td>194.248 ± 1.125</td>
<td>84.245 ± 0.486</td>
<td>0.239 ± 0.002</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Night</td>
<td>179.212 ± 13.765</td>
<td>92.783 ± 6.055</td>
<td>0.221 ± 0.012</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Prod.</td>
<td>169.720 ± 7.871</td>
<td>90.261 ± 3.814</td>
<td>0.227 ± 0.007</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Day</td>
<td>48.894 ± 0.222</td>
<td>69.416 ± 0.430</td>
<td>0.138 ± 0.002</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Night</td>
<td>49.603 ± 1.299</td>
<td>70.633 ± 2.246</td>
<td>0.136 ± 0.009</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Prod.</td>
<td>49.636 ± 0.699</td>
<td>70.599 ± 1.145</td>
<td>0.141 ± 0.004</td>
</tr>
</tbody>
</table>
Table B.2: Rates in background sections at peak 2, 25 cm above the bottom of the trap.

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$k_f$</th>
<th>Type</th>
<th>PMT 1 (Hz.)</th>
<th>PMT 2 (Hz.)</th>
<th>Coinc. (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4200</td>
<td>7612</td>
<td>Day</td>
<td>143.174 ± 2.622</td>
<td>95.490 ± 2.151</td>
<td>0.135 ± 0.038</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Prod.</td>
<td>147.231 ± 6.057</td>
<td>99.033 ± 5.208</td>
<td>0.164 ± 0.102</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Day</td>
<td>132.845 ± 2.624</td>
<td>97.734 ± 2.710</td>
<td>0.138 ± 0.037</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Prod.</td>
<td>145.603 ± 7.561</td>
<td>111.828 ± 8.993</td>
<td>0.179 ± 0.117</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Day</td>
<td>195.079 ± 8.019</td>
<td>82.673 ± 2.039</td>
<td>0.226 ± 0.047</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Prod.</td>
<td>170.009 ± 11.863</td>
<td>88.409 ± 3.979</td>
<td>0.190 ± 0.082</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Day</td>
<td>48.852 ± 1.227</td>
<td>69.475 ± 1.568</td>
<td>0.137 ± 0.034</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Prod.</td>
<td>49.843 ± 3.879</td>
<td>71.460 ± 4.015</td>
<td>0.153 ± 0.078</td>
</tr>
</tbody>
</table>

...
Table B.3: Rates in background sections at Peak 1, 38 cm above the bottom of the trap

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$k_f$</th>
<th>Type</th>
<th>PMT 1 (Hz.)</th>
<th>PMT 2 (Hz.)</th>
<th>Coinc. (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4200</td>
<td>7612</td>
<td>Day</td>
<td>144.840 ± 2.290</td>
<td>97.446 ± 2.513</td>
<td>0.134 ± 0.033</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Night</td>
<td>148.204 ± 3.443</td>
<td>99.222 ± 3.558</td>
<td>0.166 ± 0.074</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Prod.</td>
<td>147.573 ± 4.132</td>
<td>99.156 ± 3.922</td>
<td>0.136 ± 0.074</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Day</td>
<td>133.241 ± 3.269</td>
<td>100.253 ± 3.004</td>
<td>0.144 ± 0.037</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Night</td>
<td>144.316 ± 5.127</td>
<td>109.640 ± 3.813</td>
<td>0.195 ± 0.090</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Prod.</td>
<td>141.758 ± 3.866</td>
<td>109.916 ± 4.539</td>
<td>0.144 ± 0.071</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Day</td>
<td>197.283 ± 9.883</td>
<td>84.698 ± 2.342</td>
<td>0.229 ± 0.051</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Night</td>
<td>181.619 ± 5.618</td>
<td>94.574 ± 2.575</td>
<td>0.239 ± 0.079</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Prod.</td>
<td>171.336 ± 7.133</td>
<td>92.178 ± 4.738</td>
<td>0.209 ± 0.096</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Day</td>
<td>49.894 ± 1.262</td>
<td>71.256 ± 2.198</td>
<td>0.139 ± 0.072</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Night</td>
<td>50.808 ± 2.198</td>
<td>73.272 ± 2.512</td>
<td>0.139 ± 0.072</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Prod.</td>
<td>50.904 ± 2.685</td>
<td>72.810 ± 3.137</td>
<td>0.147 ± 0.074</td>
</tr>
</tbody>
</table>

Table B.4: Measured background rates in background sections during the holding time, 49 cm above the bottom of the trap

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$k_f$</th>
<th>Type</th>
<th>PMT 1 (Hz.)</th>
<th>PMT 2 (Hz.)</th>
<th>Coinc. (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4200</td>
<td>7612</td>
<td>Day</td>
<td>147.282 ± 2.484</td>
<td>99.125 ± 2.538</td>
<td>0.122 ± 0.033</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Night</td>
<td>151.324 ± 4.092</td>
<td>101.603 ± 4.017</td>
<td>0.153 ± 0.090</td>
</tr>
<tr>
<td>4200</td>
<td>7612</td>
<td>Prod.</td>
<td>147.362 ± 6.423</td>
<td>98.004 ± 4.662</td>
<td>0.126 ± 0.089</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Day</td>
<td>151.983 ± 5.017</td>
<td>105.410 ± 3.374</td>
<td>0.131 ± 0.036</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Night</td>
<td>161.237 ± 4.297</td>
<td>114.170 ± 4.347</td>
<td>0.160 ± 0.061</td>
</tr>
<tr>
<td>7612</td>
<td>9600</td>
<td>Prod.</td>
<td>156.971 ± 5.758</td>
<td>109.382 ± 5.008</td>
<td>0.122 ± 0.060</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Day</td>
<td>197.894 ± 9.509</td>
<td>85.202 ± 2.188</td>
<td>0.223 ± 0.043</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Night</td>
<td>185.025 ± 8.745</td>
<td>96.356 ± 2.785</td>
<td>0.234 ± 0.058</td>
</tr>
<tr>
<td>9600</td>
<td>13309</td>
<td>Prod.</td>
<td>169.800 ± 10.656</td>
<td>91.337 ± 4.157</td>
<td>0.224 ± 0.093</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Day</td>
<td>50.143 ± 1.369</td>
<td>71.595 ± 1.712</td>
<td>0.137 ± 0.039</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Night</td>
<td>51.957 ± 1.792</td>
<td>74.054 ± 2.002</td>
<td>0.151 ± 0.059</td>
</tr>
<tr>
<td>13309</td>
<td>15000</td>
<td>Prod.</td>
<td>50.374 ± 4.850</td>
<td>70.990 ± 5.188</td>
<td>0.132 ± 0.113</td>
</tr>
</tbody>
</table>

Based on the difference between daytime and nighttime running, the single PMT mode of running becomes untenable without additional background data. Taking the
bulk of the 2017 data, where we see a significant shift between PMT1 and PMT2 lifetimes, an additional background systematic of between $\pm 0.98$ s and $\pm 3.32$ s for PMT1 and between $\pm 0.42$ s and $\pm 4.57$ s for PMT2 must be incorporated for the single photon lifetime. In the future, running production-like background runs should reduce systematic differences between daytime and nighttime mean rates. As the dagger movement pattern is the same, the primary mechanism for a daytime-nighttime discrepancy would come from temperature and not from second-order timing issues.
The reported number of photons in coincidence events, \( \epsilon_T \), can be used as an extra parameter in the likelihood fit. This is defined as the average amplitude of \( U \) coincidence events:

\[
\epsilon_T = \frac{\gamma \mu}{U} = \frac{\sum_{i=0}^{U} \gamma_i}{U}
\]  

(C.1)

If the uncertainties on the counted number of photons follows Poisson statistics, the uncertainty for the counted number of photons is \( \delta(\gamma \mu) = \sqrt{\gamma \mu} \). A similar relationship holds for the unload counts, \( \delta(U) = \sqrt{U} \).

In order to determine the uncertainty on \( \epsilon_T \) from a combination of parameters we have already measured, one can proceed through normal uncertainty propagation on equation (C.1):

\[
\delta(\epsilon_T) = \frac{\gamma \mu}{U} \sqrt{\left( \frac{\delta(\gamma \mu)}{\gamma \mu} \right)^2 + \left( \frac{\delta(U)}{U} \right)^2 + \left( \frac{2 \delta(\gamma \mu) \delta(U)}{\gamma \mu U} \right)^2}.
\]  

(C.2)

Setting the cross term to zero and replacing \( \gamma \mu = \epsilon_T U \), equation (C.2) simplifies to:

\[
\delta(\epsilon_T) = \epsilon_T \sqrt{\frac{1 + \epsilon_T}{\epsilon_T U}}.
\]  

(C.3)

Finally, a simplifying assumption can be made to somewhat decouple the number of coincidences counted from the uncertainty on the scaling factor. Given a perfectly efficient coincidence counting algorithm, the denominator of equation (C.3) becomes the
single photon yield:

\[
\delta(\epsilon_T) = \epsilon_T \sqrt{1 + \frac{\epsilon_T}{U_{PE}}}.
\]  

(C.4)

For an inefficient coincidence counting algorithm, the uncertainty will be underestimated. However, in the case where the photon amplitude of a neutron event does not differ between high and low rates, the likelihood function involving equation (C.4) will just be multiplied by a constant. When dealing with a maximal likelihood normalization, this will not affect the lifetime other than an additional constant multiplying the resultant covariance matrix. A true likelihood function generated from equation (C.4) requires some background subtraction, which could potentially have a statistical bias. As the single photon counting method had additional background issues in this work, potential statistical biases from this efficiency scaling factor have not been investigated.
APPENDIX D
PHASE SPACE EVOLUTION AND MEAN ARRIVAL TIME

D.1 3-STEP NEUTRON COUNTING

Phase Space Evolution (PSE), as discussed in section 6.6, could potentially lead to variations in the efficiency of counting between short and long holding times. To leading order, the mean arrival time shift between short and long is sufficient to characterize a variation in detection efficiency. Assume that while the dagger is stationary, the instantaneous counting rate $R(t')$ follows a single exponential:

$$R(t') = \frac{N e^{-t'/\eta}}{\eta}. \quad (D.1)$$

For a given dagger dip, the measured draining time, $\eta$, will be related to both the neutron lifetime, $\tau_n$, and the counting time of that particular detector, $\eta^\text{det}$:

$$\eta^{-1} = \left( \frac{1}{\tau_n} \right) + \left( \frac{1}{\eta^\text{det}} \right). \quad (D.2)$$

An alternate way to describe this is to utilize the branching ratio, $BR$, of counted neutrons to decayed neutrons:

$$BR_j = \frac{1/\eta^\text{det}_j}{1/\eta_j} = \frac{1/\eta_j - 1/\tau_n}{1/\eta_j}. \quad (D.3)$$

By measuring the draining times and knowing the neutron lifetime, we can thus relate the yields $Y$ in our $j$ dips to the counting and neutron lifetimes. Defining a matrix, $M_{j-\text{dip}}$, that equates the number of neutrons $N_j$ with an energy capable of reaching the $j^{th}$ dip with the number of neutrons remaining in that dip that have not previously been
counted or decayed, the yield can be written as a matrix equation. Each dip occurs at time $T_j$ and lasts for $T_j^\Delta$. In the case of $j = 3$ dips, our nominal production running, this matrix equation becomes:

$$Y = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} M_{3-dip} \begin{bmatrix} BR_1 \int_{T_1}^{T_1 + T_1^\Delta} e^{(-t' - T_1)/\eta_1} dt' \\ BR_2 \int_{T_2}^{T_2 + T_2^\Delta} e^{(-t' - T_2)/\eta_2} dt' \\ BR_3 \int_{T_3}^{T_3 + T_3^\Delta} e^{(-t' - T_3)/\eta_3} dt' \end{bmatrix}. \quad (D.4)$$

In order to convert this to a correction, one can write the matrix $M_{3-dip}$ explicitly. Additionally, the number of neutrons in a dip, $N_j$, can be re-written as a fraction of the total number of neutrons in the trap, $N_j = f_j N_{tot}$. Expanding out the 3-dip matrix equation gives:

$$Y = e^{-T_1/\tau_n} \begin{bmatrix} f_1 N_{tot} & f_2 N_{tot} & f_3 N_{tot} \end{bmatrix} \begin{bmatrix} 1 & e^{-T_1^\Delta/\eta_1} & e^{-T_1^\Delta/\eta_1} e^{-T_1^\Delta/\eta_2} \\ 0 & e^{-T_2^\Delta/\tau_n} & e^{-T_2^\Delta/\tau_n} e^{-T_2^\Delta/\eta_2} \\ 0 & 0 & e^{-T_3^\Delta/\tau_n} e^{-T_3^\Delta/\eta_3} \end{bmatrix} \begin{bmatrix} BR_1 (1 - e^{-T_1^\Delta/\eta_1}) \\ BR_2 (1 - e^{-T_2^\Delta/\eta_2}) \\ BR_3 (1 - e^{-T_3^\Delta/\eta_3}) \end{bmatrix}. \quad (D.5)$$

An indication of PSE would manifest as a population shift where $f_2 \leftrightarrow f_3$, shifting the lifetime. Using equation (D.5), the number of neutrons counted in step 2 becomes:

$$Y_2 = (f_1 N_{tot} e^{-T_1/\tau_n} e^{-T_1^\Delta/\eta_1} + f_2 N_{tot} e^{-T_2/\tau_n}) \left[ 1 - e^{-T_2^\Delta/\eta_2} BR_2 \right]. \quad (D.6)$$

Since $f_1 \approx 0$, equation (D.6) can be used to calculate the shift in $f_2$ between different runs, and thus the change in efficiency between short and long holding times.
D.2 MEAN ARRIVAL TIME

Another method of measuring PSE involves looking at the Mean Arrival Time (MAT), $\langle t \rangle$. The MAT can be written as the time integrated sum of all the counts in our three dips, normalized to the number of counts we actually observe. Integrating over the observed rate, $R_j$, the mean arrival time is:

$$\langle t \rangle = \frac{BR_1 \int_{T_1}^{T_1+T_1^A} t' R_1 (t') \, dt' + BR_2 \int_{T_2}^{T_2+T_2^A} t' R_2 (t') \, dt' + BR_3 \int_{T_3}^{T_3+T_3^A} t' R_3 (t') \, dt'}{BR_1 \int_{T_1}^{T_1+T_1^A} R_1 (t') \, dt' + BR_2 \int_{T_2}^{T_2+T_2^A} R_2 (t') \, dt' + BR_3 \int_{T_3}^{T_3+T_3^A} R_3 (t') \, dt'}.$$  \hspace{1cm} (D.7)

An equivalent way of writing this would be to re-write the matrix form of equation (D.5), where we can explicitly solve for the integrals:

$$\langle t \rangle = \frac{e^{-T_1/\tau_n}}{Y} \begin{bmatrix} f_1 N_{tot} & f_2 N_{tot} & f_3 N_{tot} \end{bmatrix} \begin{bmatrix} 1 & e^{-T_1^A/\eta_1} & e^{-T_1^A/\eta_1} e^{-T_2^A/\eta_2} \\ 0 & e^{-T_1^A/\tau_n} & e^{-T_1^A/\eta_1} e^{-T_2^A/\eta_2} \\ 0 & 0 & e^{-T_1^A/\tau_n} e^{-T_2^A/\eta_2} \end{bmatrix} \begin{bmatrix} BR_1 \left( \left( T_1 + \eta_1 \right) \left[ 1 - e^{-T_1^A/\eta_1} \right] - T_1^A e^{T_1^A/\eta_1} \right) \\ BR_2 \left( \left( T_2 + \eta_2 \right) \left[ 1 - e^{-T_2^A/\eta_2} \right] - T_2^A e^{T_2^A/\eta_2} \right) \\ BR_3 \left( \left( T_3 + \eta_3 \right) \left[ 1 - e^{-T_3^A/\eta_3} \right] - T_3^A e^{T_3^A/\eta_3} \right) \end{bmatrix}.$$  \hspace{1cm} (D.8)

The mean arrival time will decrease as the fraction of neutrons in step 2, $f_2$, increases, so a shift in one of these measured quantities will cause a lifetime shift. There are two competing systematic effects due to the PSE of our neutrons in storage; not only are neutrons arriving at different times, but the efficiency, $\epsilon$ also changes. This will affect the measured lifetime. For a short run $S$ and a long run $L$:

$$\tau_{meas} = \frac{\langle t \rangle_L - \langle t \rangle_S}{\ln \frac{\epsilon_S N_S}{\epsilon_L N_L}} = \frac{t_L - t_S + (\Delta(t_L) - \Delta(t_S))}{\ln \frac{Y_S}{Y_L} + \ln \frac{\epsilon_S}{\epsilon_L}}.$$  \hspace{1cm} (D.9)
With phase-space evolution, the counting fraction at a later time will be shifted by some small perturbation:

$$f(t_L, \epsilon) = f(t_S, \epsilon) + \delta_f(t_L, \epsilon). \quad (D.10)$$

In the case that the perturbation $\delta_f$ is small, the yield $Y$ can be found by combining equations (D.1) and (D.3) into equation (D.5), and generically summing over the dips $j$. The yield then becomes:

$$Y = \int_{t' = 0}^{T^\Delta} Y_{all} \int_0^{T^\Delta_j} e^{-t'/\tau_n} \left( 1 - \frac{\eta_j}{\tau_n} \right) dt' = \sum_j f_j N_{tot} \int_0^{T^\Delta_j} e^{-t'/\tau_n} \left( 1 - \frac{\eta_j}{\tau_n} \right) dt' = N_{tot} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) \left[ 1 - e^{-T^\Delta / \eta(\epsilon)} \right]. \quad (D.11)$$

In equation (D.11) we have taken the continuum limit; $\sum_j N_j = N_{tot} \int f(\epsilon) d\epsilon$, with $\int f(\epsilon) d\epsilon = 1$. This is just a statement that the total fraction of neutrons, no matter the relative efficiencies, must ultimately become one. In a single-dip case, where the counts are integrated sufficiently long enough such that $T^\Delta \to \infty$, the yield becomes:

$$Y(t') = N_{tot}(t') \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon = N_{tot} e^{-t'/\tau_n} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon. \quad (D.12)$$

In the case that the perturbation is small and null over the full spectrum, such that
\[ \int \delta f(\epsilon) d\epsilon = 0, \text{ the measured lifetime becomes:} \]

\[
\tau_{\text{meas}} = \frac{(t_L - t_S)}{\ln \left( \frac{Y_S}{Y_L} \right)} \]

\[
= \frac{(t_L - t_S)}{\ln \left( \frac{N_{\text{tot}} e^{-t_S/\tau_n} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{N_{\text{tot}} e^{-t_L/\tau_n} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right)} \]

\[
= \frac{(t_L - t_S)}{(t_L - t_S) / \tau_n} \]

\[
= \tau_n. \]

Without the branching ratio for detection, \( BR = 1 - \frac{\eta(\epsilon)}{\tau_n} \), then \( \int f(\epsilon) d\epsilon = 1 \) and so the integrals in equation (D.13) cancel out the effects of any time evolution. If the neutrons in the trap take a long time to reach equilibrium, then any PSE leads to a time-varying \( f(\epsilon) \), which does not perfectly cancel out. If we now include the small time evolution perturbation, equation (D.10), in the integrals in equation (D.13), the resulting lifetime will be shifted:

\[
\tau_{\text{meas}} = \frac{(t_L - t_S)}{\ln \left( \frac{N_{\text{tot}} e^{-t_S/\tau_n} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{N_{\text{tot}} e^{-t_L/\tau_n} \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right)} \]

\[
\approx \frac{(t_L - t_S)}{(t_L - t_S) / \tau_n - \ln \left( 1 + \frac{\int \delta f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right)} \]

\[
\approx \tau_n \left( 1 + \frac{\tau_n \int \delta f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{(t_L - t_S) \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) \]

\[
\approx \tau_n \left( 1 - \frac{\int \delta f(\epsilon) \eta(\epsilon) d\epsilon}{(t_L - t_S) \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right). \]

That is, in the presence of a non-negligible \( \delta f \), there is a factor modifying the lifetime
by the ratio of the integrals of the $\delta f$ and $f$, meaning that $\tau_{\text{meas}} \neq \tau_n$. In order to simplify the last part of this, we invoked $\int \delta f(\epsilon) d\epsilon = 0$. This extra perturbation term is the shift in lifetime due to PSE. Now, we want to calculate the effect due to a change in MAT. Summing over all dips, $j$, the MAT can be written as:

$$
\langle t \rangle_S = t_S + \frac{\sum_j f_j Y_j \left( \int_0^{T_j^s} t' \frac{e^{-t'/\eta_j}}{\eta_j} d\epsilon \right) \left[ 1 - \frac{\eta_j}{\tau_n} \right]}{\sum_j f_j Y_j \left( \int_0^{T_j^s} \frac{e^{-t'/\eta_j}}{\eta_j} d\epsilon \right) \left[ 1 - \frac{\eta_j}{\tau_n} \right]}.
$$

(D.15)

In the limit of sufficiently long integration $T^A \to \infty$, this simplifies to:

$$
\langle t \rangle_S = t_S + \frac{\sum_j f_j T_j \left( 1 - \frac{\eta_j}{\tau_n} \right)}{\sum_j f_j \left( 1 - \frac{\eta_j}{\tau_n} \right)}.
$$

(D.16)

The long hold can be treated the same, but now incorporating the $\delta f$ perturbation between long and short holding times:

$$
\langle t \rangle_L = t_L + \frac{\int \left[ f(\epsilon) + \delta f(\epsilon) \right] \eta(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{\int \left[ f(\epsilon) + \delta f(\epsilon) \right] \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}.
$$

(D.17)

In a lifetime measurement, we can take the difference of $\langle t \rangle_L$ and $\langle t \rangle_S$ instead of $t_L$ and $t_S$, as in equation (D.9). In this case, the difference between equations equation (D.16)and equation (D.17) can be somewhat modified through a Taylor expansion
around a small $\delta_f(\epsilon)$:

$$
(t)_L - (t)_S = t_L - t_S + \frac{\int [f(\epsilon) + \delta_f(\epsilon)] \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int [f(\epsilon) + \delta_f(\epsilon)] \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} - \frac{\int f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}
$$

$$
\approx t_L - t_S + \left(\frac{\int f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} - \frac{\int f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}\right)
$$

$$
+ \frac{\int \delta_f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}
$$

$$
- \frac{\int \delta_f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} \frac{\int f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} + \ldots
$$

$$
\approx t_L - t_S + \frac{\int \delta_f(\epsilon) \eta(\epsilon) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} - \frac{\int \delta_f(\epsilon) \left(\frac{\eta^2(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}
$$

$$
+ \frac{\int \delta_f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} \frac{\int f(\epsilon) \eta(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon}{\int f(\epsilon) \left(1 - \frac{\eta(\epsilon)}{\tau_n}\right) d\epsilon} + \ldots
$$

(D.18)

Now if we put equation (D.18) into the lifetime measurement in equation (D.9), while
absorbing the efficiencies $\epsilon_S$ and $\epsilon_L$ into their respective yields, the lifetime becomes:

$$
\tau = \frac{\langle t \rangle_L - \langle t \rangle_S}{\ln \frac{Y_S}{Y_L}} \\
= \frac{\tau_n}{t_L - t_S} \left( \langle t \rangle_L - \langle t \rangle_S \right) \\
= \tau_n + \frac{\tau_n}{t_L - t_S} \left( \frac{\int \delta_f(\epsilon) \eta(\epsilon) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} - \frac{\int \delta_f(\epsilon) \left( \frac{\eta^2(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) \\
+ \frac{\int \delta_f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \left( \frac{\int \delta_f(\epsilon) \eta(\epsilon) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} - \frac{\int \delta_f(\epsilon) \left( \frac{\eta^2(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) + \ldots \\
= \tau_n \left( 1 + \frac{1}{t_L - t_S} \left( \frac{\int \delta_f(\epsilon) \eta(\epsilon) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) \right) \\
+ \frac{\langle t \rangle_S - t_S}{\tau_n (t_L - t_S) \int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \left( \frac{\int \delta_f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} - \frac{\int \delta_f(\epsilon) \left( \frac{\eta^2(\epsilon)}{\tau_n} \right) d\epsilon}{\int f(\epsilon) \left( 1 - \frac{\eta(\epsilon)}{\tau_n} \right) d\epsilon} \right) \ldots \\

(D.19)
$$

Here we have used equation (D.16) to simplify the multiplicative components of the expansion. Note that the use of the $\langle t \rangle$ in equation (D.19), to first order, cancels the effect of variable $\epsilon$ as in equation (D.14). The higher order terms will all be suppressed by at least a factor of $\tau_n^{-1}$. As a quick estimate of the bias introduced by the mean arrival time, the higher order terms are somewhere around $\sim 70/880 \times 0.1 = 0.0079$ s, as $\tau_n \sim 880$ s, the MAT of a short hold is $\sim 70$ s after the hold, and the PSE shift is around $\sim 0.1$ s. This is an overestimate, and well below the statistical uncertainty of calculating the PSE and the mean arrival time.
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Graduate Associate Instructor August 2015-May 2016
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- Lab Instructor for Physics 201, Introductory Mechanics for non-majors
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- Designed user interfaces for simulations and simultaneous operation of the Cornell Energy Recovery Linac
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"Measuring Systematics in the UCNτ Experiment"  
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